VEDIC MATHEMATICS

OVER VIEW

N. Puri

SPIRITUAL SCIENCE SERIES
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कृपया पुस्तक के ऊपर कोई निशान आदि
न लगायें।

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WORKSHOP ON VEDIC MATHEMATICS

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An Over View of Vedic Mathematics

by

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I LOVE MY INDIA I LOVE MY INDIA
I LOVE MY INDIA
INDIA FIRST RELIGION NEXT MYSELF LAST

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-: VEDIC MATHEMATICS SUTRAS :-

1. By one more than the previous one
2. All from nine & last from ten
3. Vertically & Crosswise
4. Transpose & Apply
5. The summation is equal to zero
6. If one is in ratio, the other one is zero
7. By addition and subtraction
8. By the completion and noncompletion
9. Sequential motion
10. The deficiency
11. Whole as one & one as whole
12. Remainder by the last digit
13. Ultimate and twice the penultimate
14. By one less than the previous one
15. The whole product is the same
16. Collectivity of multipliers

ANCIENT VEDIC MATHEMATICS

SALIENT FEATURES

16 Vedic sutras of 120 words can solve all mathematical problems.

With a firm scientific footing

A revolution in concept of learning

Opening new areas of R & D

The Vedic Maths can be used to check the Computer results, and frequently supersedes calculators

Revealing hither-to unknown, Great and Mysterious Qualities of Numbers

Simple, short mental mathematical processes often with magic speed

Requires no prerequisite courses

Mathematics otherwise burden, turns to be a joyful subject

A boon for all competitions

More useful, accurate and applicable at times, compared to modern computers

Speed of learning maths is nearly ten times

A Complete System

A most natural Vedic system, which developed the human computer to wonderful levels.

The problem solutions flow naturally to the mind with effortless efforts.

A system with mental multichoice procedures, which keep the mind alert and agile. VM has a unique independent cross checking system.

A system which children learn with smiles and most often ask, 'why Vedic Maths is not being taught in every school and college in the whole world.

THE SIXTEEN SUTRAS OF VEDIC MATHEMATICS

सूत्र :-

१ एकाधिकेन पूर्वेण	1. Ekadhikena Purvena (also a corollary)
२ निखिलं नवतश्चरमं दशतः	2. Nikhilam Navatascaramam Dasatah
३ ऊर्ध्वतिर्यग्भ्याम्	3. Urdhva-tiryagbhyam
४ परावर्त्य योजयेत्	4. Paravartya Yojayet
५ शून्यं साम्यसमुच्चये	5. Sunyam Samyasamuccaye
६ (आनुरुप्ये) शून्यमन्यत्	6. (Anurupye) Sunyamanyat
७ संकलनव्यवकलनाभ्याम्	7. Sankalana-vyavakalanabhyam (also a corollary)
८ पूरणपूरणाभ्याम्	8. Puranapuranabhyam
९ चलनकलनाभ्याम्	9. Calana-Kalanabhyam
१० यावदूनम्	10. Yavadunam
११ व्यष्टिसष्टिः	11. Vyastisamastih
१२ शेषाण्यङ्.केन चरमेण	12. Sesanyankena Caramena
१३ सोपान्त्यद्वयमन्त्यम्	13. Sopantyadvayamantyam
१४ एकन्यूनेन पूर्वेण	14. Ekanyunena Purvena
१५ गुणितसमुच्चयः	15. Gunitasamuccayah
१६ गुणकसमुच्चयः	16. Gunakasamuccayah

THE FOUNDATIONS OF VEDIC MATHEMATICS

Computational techniques are just one aspect of the complete approach of Vedic Mathematics. Maharishi's Vedic Science locates the deepest value of Vedic Mathematics in the mathematical structure of consciousness itself. The founders of the Vedic tradition discovered the capability of the human mind to settle into a state of deep silence while remaining awake, and therein to experience a completely unified, simple, and unbounded state of awareness, called pure consciousness. In this experience, Maharishi explains, the conscious mind becomes identified with an infinite, all-prevading unified field, an eternal continuum underlying all existence.

Maharishi describes the Ved, within the field of pure consciousness, as existing on the level of undivided unity. Within this unity two values, existence and intelligence, can be located. It is intelligence value that knows itself and thereby conceptually creates three values within itself-the knower, the process of knowing, and the known. Thus, one becomes two, and two becomes three. Through the further interaction of these three values, the infinite diversity of the universe is created from the sequential quantification of unity. This unfoldment is portrayed with perfect precision by Vedic Mathematics.

According to Vedic Science, all values of relationship and transformation, both abstract and concrete, are simply expression of the self-interacting structure and dynamics of an underlying, totally unified field of pure intelligence. The unlimited variety of relationships and transformations, ranging from the most abstract areas of modern mathematics to the most concrete phenomena of nature, can thereby be described in terms of the inner relationships of pure consciousness. Vedic Mathematics provides this description.

VEDIC MATHEMATICS
THE COSMIC SOFTWARE FOR THE COSMIC COMPUTER

INTRODUCTION

Vedic Mathematics offers a new holistic approach to mathematics and to mathematics education. Its range extends from the most concrete values of numerical computation to the most abstract aspects of the dynamics of intelligence.

The computational methods of Vedic Mathematics have been found to be easier to learn, faster, and more enjoyable than conventional methods. The intrinsic flexibility of the Vedic system of calculation develops the students' creativity and clearity of mind.

The system of Vedic Mathematics is derived from the Vedic tradition of knowledge. At its deepest level, Vedic Mathematics is the exploration and exposition of the self-interacting structure of pure consciousness, which is the orderly basis of all phenomena in nature. By connecting mathematics to its source in the field of pure consciousness, Vedic Mathematics has the potential to bring fulfillment to all areas of mathematics and to solve the current crisis in mathematics education.

THE SUTRAS OF VEDIC MATHEMATICS

All the computational methods of Vedic Mathematics are based on sixteen sutras (aphorisms) that are directly derived from the Atharv-Ved, one of the primary texts of the ancient Vedic literature of India. This mental computational system was brought to light in recent times by the Shankaracharya of Puri, Swami Bharati Krishna Tirtha.

On one level, each sutra aphoristically describes the actual computational method to be used for the various types of mathematical problems. On a deeper level, however, each sutra may be seen as a refined formula for producing a high degree of coherence and order in brain functioning, facilitating the rapid and precise solution to the mathematical problem.

The coherently functioning brain, with its billions of neurons and interconnections is, in Maharishi's words, a "cosmic computer". Through proper programming it can accomplish anything. The sutras of Vedic Mathematics are the "cosmic software" that create the ability to compute most rapidly and precisely.

APPLICATIONS TO MATHEMATICS EDUCATION

Vedic Mathematics offers a new approach to resolving the current crisis in mathematics education. It is not simply a collection of new computational techniques; rather, it provides an entirely different approach to mathematical computation, based on pattern recognition.

The conventional methods of mathematical computation consist of rigid, sometimes monotonous procedures that are uniformly applied to all problems of a given type. The result is often boredom, strain and a lack of enjoyment for both student and teacher.

By contrast, the techniques of Vedic Mathematics allow for constant expression of a student's creativity. First, for any type of problem, there are always several different approaches : at least one general technique, application to all cases, and also a number of special techniques, applicable to cases having a particular pattern. The general technique is always faster and easier than the conventional technique, while the techniques for special cases work extraordinarily fast, yielding solutions to complex problems, often at the speed of just writing down the successive digits of the answer. Second, at each stage of a particular technique, there are usually options with regard to the next step. This flexibility in the method keeps the mind lively and alert and cultures the ability to quickly discover the path of least action on the way to the solution.

In addition, the techniques of Vedic Mathematics have been found to be easier to learn than conventional methods. Computational complexities are significantly reduced by using digits and their complements. And finally, unique, independent cross-checking methods make the system less prone to error.

In summary, then, the techniques of Vedic Mathematics not only provide a quick and accurate solution to the problem at hand, but also develop clarity of mind and intuition In this way, Vedic Mathematics succeeds in fulfilling the two major goals of mathematics education : to teach students procedures for solving specific mathematical problems and to develop clear and logical thinking.

The problem of "math anxiety" which confronts mathematics educators around the world, simply does not arise. The Vedic Mathematics research group in London has successfully applied Vedic Mathematics at both primary and secondary school levels. The Indian Ministry of Human Resource Development recently held a workshop on Vedic Mathematics at Rajasthan University, which was attended by mathematicians, educators, and government representative. The workshop recommnended that the government

should set up a national committee to followup further work in Vedic Mathematics and plan its implementation in the teaching in schools and colleges. Further, the government funding agencies like University Grants Commission etc. should be asked to provide funds for further research and development of Vedic Mathematics. The national committee is already started organising series of workshops on Vedic Mathematics. These workshops are being organised by the Rashtriya Ved Vidya Pratishthan.

The general principle is familiar to modern mathematicians. The unfied foundation of modern mathematics is provided by set theory, the mathematics of the infinite. At the foundation of set theory are mathematical principles that directly describe the nature of an abstract, infinitely dynamic field of intelligence. All of mathematics is described as sequentially unfolding from the internal dynamics of this field of pure intelligence. For example, all sets are sequentially created from the unmanifest point value of the null set. This self-referral nature of the field of intelligence at the basis of set theory is expressed at a deeper level by the reflection principle, which underlies the mathematical description of the different quantified values of infinity provided by the theory of large cardinals.

In Vedic Mathematics, all the fundamental principles are derived from direct experience of the unmanifest source of orderlines within the unifield field of pure consciousness. Vedic Mathematics is, therefore, the mathematics of the most fundamental structures of natural law. This is its unique stature and contribution to the field of knowledge.

INVITATION

The computational applications of Vedic Mathematics are only the first step in an ongoing research program. All areas of modern mathematics, both pure and applied, fall within the scope of Vedic Mathematics. We are just beginning to tap the enormous potential of this more fundamental and complete approach to knowledge. All are invited to join the research programme.

VEDIC MATHEMATICS SYSTEM

Whatever we see in nature has a perfect mathematical structure. The rotation of the earth, movement of the stars and planets, growth of the plants and trees etc. are all perfectly mathematically structured. The blooming of a flower, the growth of a plant, the rotation of the earth, all go on so very effortlessly. Recent research is bringing to light the perfect mathematical structure of the Vedic texts. As a matter of fact whatever we do in life has mathematics in some form or the other. It is not possible to prepare even a delicious cup of coffee without a precise sense and application of mathematical proportion of different ingredients used. It's a pity that such a wonderful subject of fundamental and vital importance 'mathematics' is creating the problem of 'MATHS ANXIETY' in all parts of the world. The common students are afraid of Mathematics. The gravity of the problem is quite clear from a recent survey report published in America. The top 10% of the students of Maths were asked the question, "Do you like Mathematics", And 60% of them said No. The answer of the remaining 90% is quite obvious.

Further, whatever we want to do, we always have a large number of alternatives and choices in doing the same. How strange that in Mathematics there seems to be almost no choice, and, most of the time we have to follow a unique, fixed, monotonous, often boring and tedious procedure in whatever arithmetical operation we want to do. Interestingly enough this unique approach is considered to be a quality by some mathematicians. Anything fixed, naturally becomes monotonous and boring and Maths Anxiety is the natural result. Vedic Maths on the other hand with its multiple choice makes maths a playful and interesting subject, which naturally brings frequent smiles on the face and joy in the heart of the students.

The sixteen Vedic sutras provide a structured system which is applicable in, and, hence, is capable of explaining almost any facet of cosmos, both at abstract as well as applied level. These sutras in their philosophical interpretation even explain the functioning of human psychology. In their simple applied form, their crude translation in English has been presented in the beginning.

Sanskrit is a rich language and for deeper meaning one has to dive deep into the original sutras. The first Vedic Sutra Ekadhikena Purvena in simple English means by one more than the previous one, and a couple of very interesting and wonderful applications are presented in Pushp I. However the same sutra EkadhikenaPurvena(एकाधिकेन पूर्वेण) explains the whole concept of growth and development of an individual, society or any facet of knowledge. The word purvena essentially means the previous state. Ekadhiken means one higher than (the previous state). The higher state again becomes the previous and we go higher and higher and higher step by step, and that speaks out the whole process of growth and development. Similarly each sutra has a deep philosophical meaning. However, in the initial Pushps we are only looking at the simple applied facets of these sutras.

Maths with Smiles

Vedas are the source of all knowledge and of all Sciences. They are not only the oldest but also the best. The Vedas have guided millions of aspirants on the path of knowledge. With the passage of time most of the Vedic knowledge has been scattered and shattered. Vedic Mathematics is no exception to it. It is inspiring to observe that, deep interest is being taken, the world over in reviving the Vedic Sciences.

According to the brain specialists the left half of the brain systematically collects the information, does sequential analysis and arrives at the logical conclusions and results. Only left half of the brain is activated and developed by the present Maths education, being given in the schools and colleges. The right half of the human brain works with pattern recognition and has the capability of intuition. This most powerful facet of human personality the intuitive faculty, unfortunately, remains undeveloped in most of the students. In Vedic Mathematics system, the very first step is to recognize the pattern of the problem and pick up the most efficient Vedic algorithm. Further, at each subsequent step, we recognize the pattern and complete the task by using the appropriate superfast, mental working procedure. As in Vedic Maths we have multiple choice available at each stage of working. This practice, systematically develops the right half of the brain as well. As such by the regular practice of the most natural mental procedures of Vedic Maths system, the **HO-LISTIC DEVELOPMENT OF THE HUMAN BRAIN** automatically takes place. In a nutshell we can say that the Vedic Maths provides the cosmic software for the cosmic computer (the human mind), so that the later is not wasted.

IN A NUT SHELL

Vedic Maths (VM) offers a new and entirely different approach to mathematics based on **pattern recognition** for resolving the current crisis in Maths education. VM allows for constant expression of a **student's creativity** and has been found to be **easier to learn.** For any problem there is always one general technique applicable to all cases. There are also number of special techniques for special pattern problems. The element of choice and flexibility at each stage keeps the mind lively and alert and develops **clarity** of **mind** and **inuition**, and, thereby Holistic development of the human brain automatically takes place.

SWAMI BHARATI KRISHNA TIRTHA

Swami Bharati Krishna Tirtha, author of the first recent Vedic Mathematics book was the Shankaracharya of Goverdhan Peath, Jagadguruji, named Venkatraman in his early days, was an exceptionally brilliant student and invariably won the first place in all the subjects in all the classes throughout his educational carrier.

In 1904, at the age of just twenty, he passed M.A., examination in seven subjects simultaneously securing the highest honours in all, which is perhaps the all time world-record of academic brilliance. His subjects included Sanskrit, Philosophy, English, Mathematics, History and Science.

In the words of Swami Ji, "These 16 Sutras (aphorisms) of barely 120 alphabets, apply to and cover each and every chapter of each and every branch of mathematics (including arithmetic, algebra, geometry - plane and solid, trigonometry - plane and spherical, conics - geometrical and analytical, astronomy, calculus etc.,). In fact, there is no part of mathematics, pure or applied, which is beyond their jurisdiction The Sutras are easy to understand, easy to apply and easy to remember; and the whole work can be truthfully summarised in one word "mental" .

Vedic Mathematics has originated from that Ved.Ved is all Divine knowledge. Any knowledge derived from the Ved is bound to have the touch of the divine bliss. Therefore the very natural, easy and superfast algorithms of Vedic Mathematics bring an upsurge of joy and bliss. Due to its very special and universal features, Vedic Maths system converts the dry and tedious Maths into a playful and joyful subject, which children learn with smiles. Therefore, the Vedic Maths is the gift of the Ved to solve the problem of "MATHS ANXIETY" being faced in maths education, in the whole world.

WORK AT ROORKEE

I wish to draw your kind attention to another facet of the humble research effort at the university campus. Everybody who has known a little about the Vedic Mathematics (VM) has derived national pride and shown a great zeal to learn more. VM kindles in our hearts, the Jyoti of 'shradha' towards Mother India.

The Spiritual Study Group of Roorkee University has been spreading this wonderful mathematical facet of ourrich, vast and scientific cultural heritage. On repeated requests since May 85, SSG, has already organised the VM courses for 41 batches attended by 4350 participants, ranging from school children, teachers, university students, research scholars, field engineers to the university professors, of USA Canada, Mexico, Holland, France, England, Germany, Switzerland, Spain etc. by Aug. 1988.

Since Haridwar Maha Kumbh, 1986, SSG has been conducting VM Correspondence Courses in Hindi & English for home study and new batches start from Dushera (October) and Baishaki (April) every year.

Thousands of Vedic Mathematics demonstration lectures are being delivered in different parts of the world. In India, hundreds of lectures have already been given in Delhi, Punjab, Gujrat, West Bengal, U.P., M.P., Kerala, A.P., Goa, Rajasthan, Hariyana and Maharashtra. Further, demonstration lectures and seminars on Vedic Mathematics have already been delivered in several cities of America, England, Canada, Mexico, Norway, Sweden, Finland, Denmark, Hungry, Germany, Italy, Switzerland, Spain, Holland in the recent past..

SSG Publications

In order to provide comprehensive details of this wonderful, ancient Bhartiya Cultural Heritage SSG is publishing a series of books in Hindi and English.

SYMPOSIUM AND WORKSHOPS

A Symposium on Vedic Mathematics was held in Feb. 1985 at Nagpur. The Second at Bhartiya Vidya Bhawan's London Centre in Sept. 1986. The first 11-day workshop on Vedic Maths, and its computational potentialities was held at University of Roorkee in Aug. 1987. A 3-day workshop was organised in Dec. 1987 by SSG at Banaras Hindu University, Varanasi, followed by a 4-day workshop in Feb. 1988 at G.B. Pant University, Pantnagar. Further, Vedic Maths workshop was organised by the Ministry of Human Resource Development and Rashtriya Ved Vidya Pratishthan at Jaipur in March and at Ahmedabad in October 1988.

Recently, Vedic Maths seminars have also been held during April, 1988 at Imperial College, London ; Oxford University, Cambridge University, and Ideal Village, Lancashire ; Washington ; Harward University, Scientific Systems, Boston ; Annual NCTM meeting at Chicago ; Los Angles ; Stanford University, Pola Alto ; University of California, Berkeley ; Indiana ; Goldendome MI University Fairfield ; Toronto ; Mexico University, MIU Oslo, Academy at Lily Hamer, Norway ; Upsala University and Stockholm University, Sweden ; Turku University, Finland ; Copenhagen University and HC Oersted Institute, Denmark ; Hamburg University and Technical University Berlin ; Institute of Leadership Budapast, Hungry. Further during the month of May, 1988 Vedic Maths seminars were held at Main University Vienna, Austria, TM Centre Rome ; Engineering Faculty University of Naples and University of Milan, Italy ; MERU Seelsberg, Geneva University Switzerland ; Paris University France ; Barcelona, Spain, MERU Vlodrop, Amsterdam University, Holland. International two week foundation courses were organised at MERU Switzerland, Washington, Mexico and MIU, IOWA USA during June-July, 1988.

Ph.D. thesis on Vedic Mathematics just submitted by Mr. S.K. Kapoor.

ILLUSTRATIONS

1. NIKAILAM SUTRA

This is the second sutra. Using this sutra we can very conveniently perform multiplication of numbers consisting of very large or small digits. In this process the Vedic Base System is used.

The simple meaning of this sutra (No. 2) is, 'All from 9 and last from 10'. This sutra is used to straight away write down the deviation of the numbers from the base for the numbers smaller than the base. Let us take an example.

Illustration 1 : Say the number is 9856. The base (i.e. the multiple of 10) close to the number 10000. The Nikhilam sutra which reads Nikhilam Navtashcaraman Dashatah tell us to substract each digit of the number from 9 and the last digit from 10.

Therefore for given number 9856 the digit by digit deviation is obtained as

9 - 9=0, 9 - 8 = 1, 9 - 5 = 4 and the last digit of the deviation is 10-6 = 4. Hence the deviation is -0144.

Similarly the deviation of 937 is -063 (Ref. 1,2 & 9).

2. VINCULUM

The vinculum number consists of both positive and negative digits.

The Approach : In the Vinculum process we take the complement of the digits bigger than 5 with a negative sign, and a positive carry over to next higher order digit.

Illustration 2 : Let us take the number 9, 9 = 10-1, therefore 9 is 1 less then 10. This we can also write as $1\bar{1}$ instead of 9. The property of the numbers is unchanged and we can add, multiply etc. in the usual manner. However, we have to keep in the mind that the $\bar{1}$ is always appearing as -1.

In the reverse order, when we need to normalise,

$1\bar{1} = 10 - 1 = 9$

Illustration 3 : 29 = 30 - 1

This can be written as $3\bar{1}$.

Illustration 4 : 3789

By Nikhilam Sutra (All from nine and last from ten), the complements of the last three digit are 9-7 = 2, 9-8 = 1, 10-9 = 1, and the previous digit gets increased by 1 (3+1 = 4). Therefore,

$3789 = 4\overline{211}$

Illustration 5 : 286923

The complement of the larger digits (869) by Nikhilam Sutra is 131. Therefore,

$286923 = 3\bar{1}\bar{3}\bar{1}23$

Note that the last digits 2 and 3 at the tenth and the unit place stay on as before, and the digit previous to the bigger ones gets increased by 1.

Illustration 6 : 353782

$353782 = 354\bar{2}\bar{2}2$

(The complement of 78 is 22)

Illustration 7 : 3728193

$3728193 = 4\bar{3}3\bar{2}2\bar{1}3$

The application of the Vinculum process should be well practised before we use it. However, its use makes the computation process quite easy and fast.

Further practise shall give us the grip on the this system, which is immensely useful for simplifying the computation processes.

Illustration 8 : $\bar{4}35\overline{27}2$

Here the digit is negative, hence it is a negative number. We may either convert all digits to bar digits, or all to + ive digits

$\bar{4}35\overline{27}2 = \overline{365268} = -365268$

or $\bar{1}634732 = -1000000 + 634732 = -365268$

At each stage we use Nikhilam Sutra.

Note : The convertion to Vinculum form and normal can be checked by using sutra 15. (Ref. 1, 2 & 8)

3. TABLES

The Vedic method gives us the key which works superfast not only upto tables of 20 but is equally efficiently applicable to any table even upto 1000 or more. (Ref. 1,2 & 9).

3.1 Operators

The operators of any number can be obtained by simply converting the given number into the Vinculum form, so that finally no digit is bigger than 5.

Therefore $19 = 2\bar{1}$; $2\bar{1}$ are the operators at the tenth and the unit places respectively.

Similar $89 = 1\overline{11}$

Therefor $\bar{1}$, $\bar{1}$, 1 are the operators at the unit place, tenth place and hundred place respectively.

The Nikhilam sutra can be used for converting any number to its Vinculum form. We can also get the Vinculum form by adding the same number to itself and than subtracting the given number from the sum without borrowing the digits from the higher place.

4. MULTIPLICATION

Nikhilam Method

It is interesting to note that there is a general formula which can be applied to all cases of multiplication. This formula is quite easy and simple. However, there are special cases where special Vedic procedure gives much simpler, easier and quicker solutions. The latter shall be discussed first. (Ref. 1, 2, 4 & 9).

Let us take the simple example of multiplying 13 by 11.

Step 1 : We write these numbers in the Vedic base system style and get :

$$\begin{array}{l} 13+3 \\ 11+1 \end{array}$$

We note that the base is 10 and there is only zero in the base. Positive sign indicates that both numbers are greater than the base.

Step 2 : The product will have2parts. A vertical dividing line should be drawn to demarcate these two parts.

$$\begin{array}{l} 13+1 \\ 11+1 \\ \hline \quad / \end{array}$$

Step 3 : We vertically multiply the excess digits from base. This product gives us the right hand side portion of the answer. In this case 3 x 1 = 3 is the product.

$$\begin{array}{l} 13+3 \\ 11+1 \\ \hline \quad /_3 \end{array}$$

Step 4 : The left hand side of the answer can be arrived at by simple adding diagonally any of the two given numbers with the deviation (deficit) of the other number from base.

Case 1 : The first number 13 is added to 1, the deficit of second number. Therefore

$13 + 1 = 14$

Case 2 : The second number 11 is added to 3 the deficit of the first number. Therefore

$11 + 3 = 14$

Note : Both of these alternatives gives the same result

13 + 3
11 + 1

14 / 3

In addition to the above two alternatives the left hand side can also be obtained by two additional procedures.

Note :

The Vedic System has this special feature of availability of the same result in several easy ways. This feature can be utilised by the student to verify his answer step by step.

Further, **Illustration 9 :**

111 x 112

111 + 11
122 + 12

$123 \,/\, {}_{1}32$ = 123+1/32 = 124/32 = 12432

Note : The R.H.S. should be obtained mentally by using Vedic method i.e.

11 + 1
12 + 2

13 / 2

10010 x 10101

10010 + 10
10101 + 101

10111/1010

1099 x 1001

1099 + 99
1001 + 01

1100 / 099

103 x 13

Taking base as 10

103 + 93
13 + 3

$106 \,/\, {}_{27}9$ = 106+27/9
= 133/9
= 1339

985 x 995

985 - 15
995 - 5

980 / 075

99976 x 99998

99976 - 24
99998 - 2

00074/00048

$$\begin{array}{l} 103 + 3 \\ \;\;97 - 3 \\ \hline 100 \,/\, 0\overline{9} \;=\; 99/100-09 \;=\; 9991 \end{array}$$

$$\begin{array}{l} 1021 + 21 \\ \;\;996 - 4 \\ \hline 1017 \,/\, 0\overline{8}\overline{4} \;=\; 1016/1000-84 \;=\; 1016916 \end{array}$$

5. NIKHILAM COROLLARIES

The previously illustrated examples were all solved by using the Nikhilam Sutra.

However, there are a few more corollaries arising out of the Nikhilam Sutra. We shall now learn these corollaries . (Ref. 1,2,4 & 9).

5.1 The First Corollary : The first corollary of Nikhilam Sutra is Yavadunam Tavadunikrtya Varganca Yojayet. It means what ever the extent of its deficiency lessen it further to that very extent; and also set up the square (of that deficiency).

This sutra essentially deals with squaring of the numbers.

Illustration 10:

$$\begin{array}{lllllll}
92^2 & = & (92 - 8)/8^2 & = & 84/64 & = & 8464 \\
94^2 & = & (94 - 6)/6^2 & = & 88/36 & = & 8836 \\
95^2 & = & (95 - 5)/5^2 & = & 90/25 & = & 9025 \\
96^2 & = & (96 - 4)/4^2 & = & 92/16 & = & 9216 \\
97^2 & = & (97 - 3)/03^2 & = & 94/09 & = & 9409 \\
989^2 & = & (989-11)/011^2 & = & 978/121 & = & 978121 \\
9989^2 & = & (9989-0011)/0011^2 & = & 9978/0121 & = & 99780121 \\
9993^2 & = & 9986/0049 & = & 99860049 & & \\
112^2 & = & 112+12/12^2 & = & 124/{}_{1}44 & = & 12544 \\
1011^2 & = & 1011+11/011^2 & = & 1022/121 = & 1022/121 & = 1022121
\end{array}$$

COMMENT : The algebraic proof of these methods have also been presented by Jagatguru Swami Sri Bharti Krisna Tirthaji Maharaja in his book on Vedic Mathematics.

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5972

5.2 The Second Corollary : The second corollary is the Sutra Ekadhikena Purvena. This sutra is used as a subsutra in the present case. This is a special case of the first corollary, and is applicable for squaring the number having 5 as the last digit. It's meaning is by one more than the previous one.

Square of 35 : Here the previous digit is 3. The sutra in this context tells us to multiply the previous digit (here 3) by one more than the previous number (3 + 1 = 4). Therefore, 3 x 4 = 12 is L.H.S. portion of the answer. The R.H.S. is simply the vertical multiplication product (5 x 5 = 25). Therefore $35^2 = 3 \times 4/5 \times 5 = 1225$.

Illustration 11 :

$$
\begin{array}{lclclcl}
65^2 & = & 6 \times 7/25 & = & 42/25 & = & 4225 \\
95^2 & = & 9 \times 10/25 & = & 90/25 & = & 9025 \\
115^2 & = & 11 \times 12/25 & = & 132/25 & = & 13225 \\
135^2 & = & 13 \times 14/25 & = & 13 + 3 \;/ & & \\
 & & & & \underline{14 + 4} \;/ \; 25 & = & 18225 \\
 & & & & 18\;/2 \;\;/ & & \\
195^2 & = & 19 \times 20/25 & & & = & 38025 \\
155^2 & = & 15 \times 16/25 & = & 240/25 & = & 24025
\end{array}
$$

5.3 The Third Corollary : In mathematical astronomy we frequently require the multiplication of multiplier digits consisting entirely of nines with other numbers (multiplicand). For multiplication of this type the subsutra Ekanyunena-Purvena (Sutra No. 14) is used .

Multiply 6 x 9

According to the sutra the multiplicand has to be decreased by 1 (this gives the LHS) and for the R.H.S. subtract the L.H.S. from the multiplier. (Get compliments of nines). Therefore 6 x 9 = (6-1)/9-(6-1) = 5/4 = 54

Illustration 12 :

19 x 99 = 18/(99-18) = 18/81 = 1881

778 x 999 = 777/222 = 777222

9863 x 999 = 9862/0137 = 98620137

3728387x9999999 = 3728386/6271613 = 37283866271613

6. DIVISION-NIKHILAM METHOD (Sutra 2) (Ref. 1,3,4 & 9)

Illustration 13:

- Divide 216 by 89

89 / 216

Step 1 : Using Nikhilam we modify the divisor

Here MD = 1 1

Step 2 : We write as 89 / 216

MD 11

Step 3 : We draw a slash after 2 digits, and number the digits of the Dividend from R.H.S. and digits of Divisor are numbered from L.H.S.

3rd 2nd 1st
89 / 2 / 1 6
1 1
1st 2nd

Step 4 : Multiphy the last digit of Dividend with the first and second digits of MD respectively and place the product below the previous digits (2nd and Ist column respectively) as follows

89 / 2/1 6
11 2 2
2/

Step 5 : On adding the digits of 2nd and Ist column we get result.

8 9 / 2/1 6
1 1 2 2
2/3 8

So Quotient = 2,

Remainder = 38

7. RECURRING DECIMALS (Sutra 1) (Ref. 1,2,4 & 9)

The Operator : The operator key number for computing the value of the recurring decimal can be obtained by applying the simple 'Ekadhikene Sutra'. The simple meaning of the Sutra 'Ekadhikene Purvena' is 'by one more than the previous one'. Thus for $\frac{1}{49}$ the operator key is one more than the previous digit (4). Therefore operator is 4+1 = 5 similarly for $\frac{1}{39}$ the operator = 3+1 = 4. For $\frac{1}{29}$, operator = 2+1 = 3. For $\frac{1}{19}$ operator = 1+1 = 2 etc.

THE PROCEDURE

'Ekadhikena Sutra' tells us that 'By one more than the previous one'. The first word' by is used both for multiplying and dividing. It is interesting to learn that this operator key can be used in both ways to compute the value of the recurring decimal. Here, we shall learn the method of using the operator key as a constant uniform multiplier.

Illustration 14 : Let us take the fraction $\frac{1}{39}$. In the result the last right hand most digit shall be 1 as the RHS digit is 9 and product of this RHS digit with the RHS digit of result is equal to 9, for recurring numbers. Further, the operator key is 3+1 = 4. The subsequent, step by step procedure is outlined below.

Step 1 : Put 1 as the extreme right digit and put a dot over it to indicate that it is the end of the set of recurring digits.

Step 2 : We multiply this digit by the operator and write down the results towards the left hand of the previous digit. If the resulting number has more than one digit we put down the extra digit below the next level. In this case 1x4 = 4 is written on the L.H.S. of 1 (the previous digit).

Step 3 : We multiply the previous digit (4) by the common constant operator key number (here 4). Therefore 4x4 = 16. We write down 6 on the LHS of previous digit 4 and excess digit 1 in the following manner (below the next level).

Step 4 : We multiply the previous digit $({}_{1}6\ 4\ 1)$ by the operator 6x4 = 24 and add carry over digit 1 to yield the result 24+1 = 25. Once again we write 5 on LHS of the previous digit, 2 below the next level.

$$\begin{array}{l} \ \ 5\ 6\ 4\ 1 \\ 2\ 1 \end{array}$$

Step 5 : We multiply the previous digit (5) by the operator, 5x4 = 20 and add carry over digit 2 to the result 20+2 = 22. The result is written as before.

$$\begin{array}{l} \ \ 2\ 5\ 6\ 4\ 1 \\ 2\ 2\ 1\ 0 \end{array}$$

Step 6 : We multiply the previous digit 2 by the operator (4) therefore 2x4 = 8 and add the carry over digit 2 to 8, so 2+8 = 10. So our result wil become

$$\begin{array}{l} \ \ 0\ 2\ 5\ 6\ 4\ 1 \\ 1\ 2\ 2\ 1 \end{array}$$

Step 7 : We multiply 0x4 = 0, and the carry out digit 1 so that 0+1 = 1 etc. So we find that we have started getting the same digits after 0. Therefore we can put a dot over 0 to indicate that the set of recurring decimals starts from 0. The decimal is placed before 0. Thus the set of recurring decimal is -

$$\frac{1}{39} = 0.\dot{0}2564\dot{1}$$

Duscission : Since 39 is a prime number, the behaviour of $\frac{1}{39}$. is different from other recurring numbers.

8. THE GENERAL METHOD OF MULTIPLICATION

So far we have seen the use of the Vedic Sutras for the solution of specific types of problems. Now we shall learn the simple Vedic general method of multiplication. This method is based on the Urdhava-Tiryagbhyam Sutra. The simple meaning of this Sutra is 'vertically and crosswise'. But this Sutra is very powerful and applicable for solving a large class of problems in mathematics. Further, it is interesting to point out that this brief Sutra has many fold applications. Prof. Nicholas, Pickles and Prof. Williams have written a full book on applications of this Sutra (see VM news). The step by step process is outlined in the following sections. (Ref. 1,2,3,4 & 9).

Illustration 15 : Let us consider the multiplication of 2 by 2 digit numbers.

Multiply 13 x 21

$$\begin{array}{r} 21 \\ \times\ 13 \\ \hline 3 \\ \hline \end{array}$$

Step 1 : We simplify the first column

$$R_1 = U_1 \times L_1$$
$$= 1 \times 3 = 3$$

$R_1 = 3$ is the extreme right part of the answer and is written below the first column.

Step 2 : We operate the first column with each of the subsequent column. We include the second column which on simplification gives the next digit

$$R_2 = U_1 \times L_2 + U_2 \times L_1$$
$$= 1 \times 1 + 2 \times 3 = 7$$

$R_2 = 7$ is the result at the next level and is written at the tenth level.

With this we complete the operation of the first column.

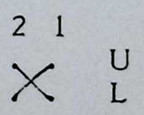
Step 3 : We eliminate the first column and simplify the remaining columns (here only one column is left) therefore

$$R_3 = U_2 \times L_2$$
$$= 2 \times 1 = 2$$

$R_3 = 2$ is the resulting digit at the next level (hundred in this case). The result is written into appropriate level.

$$\begin{array}{r} 2\ 1 \\ \times\ 1\ 3 \\ \hline 2/7/3 \end{array} = 273$$

Illustration 16 :



Multiply 79 by 68

$$\begin{array}{r} 7\ 9 \\ 6\ 8 \\ \hline {}_7\ {}^2 \end{array}$$

Simplify the first column R1 = U1 x L1 = 8 x 9 = 72

R1 = 72 is the extreme right part of the answer, consisting of two digits. Therefore, we write 2 below the first column and excess digit 7 below the next space.

Operation of the first column with the second column gives us the next part of the product

$$
\begin{aligned}
R2 &= U1 \times L2 + U2 \times L1 \\
&= 9 \times 6 + 7 \times 8 = 54 + 56 = 110
\end{aligned}
$$

Once again we enter 0 at the next space and 11 below the subsequent spaces.

$$
\begin{array}{r}
7\ 9 \\
\underline{6\ 8} \\
0\ 2 \\
117\ \
\end{array}
$$

With this we complete the operation of the first colunm. After operating the first column with all other columns we start eliminating, the column one by one.

Eliminating first column, we are left with only the second column. Therefore $R3 = U_2 x L_2 = 7x6 = 42$ is the last part of the answer. We enter this value and add the carry over digits to get the final answer

$$
\begin{array}{r}
7\ \ 9 \\
\underline{6\ \ 8} \\
4\,2\,0\,2 \\
\underline{1\,1\,7\ \ } \\
5\,3\,7\,2
\end{array}
$$

Vinculúm : Alternatively we may convert this number into the Vinculum form and then multiply

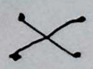
$79 = 1\ \bar{2}\ \bar{1}$ and $68 = 1\ \bar{3}\ \bar{2}$

$$
\begin{array}{r}
1\ \bar{2}\ \bar{1} \\
\underline{1\ \bar{3}\ \bar{2}} \\
\underline{3\ /\ 7\ /\ 2}
\end{array}
$$

$R1 = U_1 x L_1 = \bar{1} \times \bar{2} = 2$

$R2 = U_1 x L_2 + U_2 x L_1 = \bar{1} \times \bar{3} + \bar{2} \times \bar{2}$

$= 3 + 4 = 7$

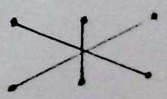


$R3 = U_1 x L_3 + U_3 x L_1 + U_2 x L_2$

$= \bar{1} \times 1 + 1 \times \bar{2} + \bar{2} \times \bar{3}$

$= \bar{1} + \bar{2} + 6$

$= 3$



With this we complete the operation of the first column with the remaining columns. For subsequent parts of the answer we start eliminating the columns, one

by one. Eliminating the first column we simplify the remaining column.

Therefore, R4 $= U_2 \times L_3 + U_3 \times L_2$

$= \bar{2} \times 1 + 1 \times \bar{3}$

$= \bar{2} + \bar{3} = \bar{5}$

and R5 $= U_3 \times L_3 = 1 \times 1 = 1$

$$\begin{array}{r} 1\ \bar{2}\ \bar{1} \\ 1\ \bar{3}\ \bar{2} \\ \hline 1/\bar{5}/3/7/2 \end{array}$$

$= 1\bar{5}372 = 5372$

Symbolically we can say that

R1 $= \begin{array}{cc}1 & \bar{2}\\ 1 & \bar{3}\end{array}\left[\begin{array}{c}\bar{1}\\ \bar{2}\end{array}\right]$; R2 $= \begin{array}{c}1\\ 1\end{array}\left[\begin{array}{cc}\bar{2} & \bar{1}\\ \bar{3} & \bar{2}\end{array}\right]$

R3 $= \left[\begin{array}{ccc}1 & \bar{2} & \bar{1}\\ 1 & \bar{3} & \bar{2}\end{array}\right]$; R4 $= \left[\begin{array}{cc}1 & \bar{2}\\ 1 & \bar{3}\end{array}\right]\begin{array}{c}\bar{1}\\ \bar{2}\end{array}$

R5 $= \left[\begin{array}{c}1\\ 1\end{array}\right]\begin{array}{cc}\bar{2} & \bar{1}\\ \bar{3} & \bar{2}\end{array}$

For each part we simplify the columnswritten within the brackets.

Notes : (1) We clearly observe that the arithmetical work involved is greatly simplified by using Vinculum approach.

(2) The whole process can be carried out mentally after a little practice.

8.1 Direct Compound Multiplication

In the Vedic method, we can do the compound multiplication directly by using the 'UrdhavaSutra'.(Ref. 1,3,4 & 9).

Illustration : Let us say we want to find the area of a plot of size 4 yards 1 feet by 8 yards 2 feet.

$$\begin{array}{lll} 4\text{ yd} & & 1\text{ feet} \\ 8\text{ yd} & & 2\text{ feet} \\ \hline 32 & /\ 16 & /\ 2 \end{array}$$

Step I : We use the Urdhava Sutra (No. 3) to obtain three parts of the answer.

(1) R.H.S = vertical product of RHS column = 2 x 1 = 2

(2) Middle part = cross multiplication = 1 x 8 + 2 x 4 = 16

This is entered in the middle portion.

(3) L.H.S. = vertical product of LHS column = 4 x 8 = 32

Step II : Now we split the middle part into two. First part consists of that number which when divided by the unit size gives the maximum whole number quotient (here 1 yard = 3 feet, therefore 3 is the unit size). And the second part is the remainder which is always less than the unit size.

In the present example 5 x 3 = 15 is the first part (which gives the maximum whole number quotient = 5) and 16 - 15 = 1 is the second part 32/16/2 = 32/5x3+1/2

Step III : In the first part we drop the unit size (here 3) and carry the multiplier (here 5) to LHS and add it to the value already there.

Here, we carry 5, therefore 32+5 = 37 square yards.

The second part (which is the remainder) is multiplied by unit size and the product carried to RHS. This is added to the value already there.

Here 1x3 = 3 is carried to RHS and added to the previous value = 2, which is already there. Therefore 2+3 = 5 square feet is the answer.

32 / 5x3 + 1 /2 = 32+5/ /1x3 + 2 = 37 square yards / 5 square feet

This example explains the simple Vedic method of compound multiplication without the use of any conversion which is essential in present method.

9. DIVISION - By Paravartaya Method (Ref. 1,3,4 & 9)

We have seen how Nikhilam Method can be applied for division by real big digits but however this is not at all suitable for division by smaller number. For division by smaller number - an equally easy and superfast vedic method is available.

Algebraic Division (Ref. 1,3,4 & 9)

Introduction : In this method of division we use the same Vedic formulae Paravartaya Yojayet, which means, 'transpose and apply'. It is interesting to observe that the Vedic method of division for algebraic equations is exactly similar to that for the numerical division as already explained in the previous section. As a matter of fact in Vedic method it seems difficult to draw a line where arithmetic stops and algebra starts. The following examples shall bring out the points of similarity and differences.

Illustration 17 : Divide $4x^2 + 15x - 13$ by $x + 1$

$$\begin{array}{l|lll}
x+1 & 4x^2 & +15x & /-13 \\
\hline
-1 & & -4 & \\
 & & & -11 \\
\hline
 & 4 & +11 & /\ -24
\end{array}$$

$Q = 4x + 11$, $R = -24$

Illustration 18 : Divide $x^4 - x^3 + x^2 - 3x + 4$ by $x^2 - x + 1$. Here we operate the two coefficients at each stage of division.

$$\begin{array}{l|lllll}
x^2-x+1 & x^4 & -x^3 & +x^2 & /-3x & +4 \\
\hline
1\ -1 & & 1 & -1 & & \\
 & & & 0 & 0 & \\
 & & & & 0 & 0 \\
\hline
 & x^2 & +0x & +0 & /-3x & +4
\end{array}$$

$Q = x^2$, $R = -3x + 4$

Notes : Since the divisor has the x^2 term we can therefore have the remainder with x terms, as such we draw the oblique line, demarcating the remainder portion before the x term of the dividend.

2. Please note that we do not modify the power of x in the remainder terms.

3. To avoid error we also write even the row consisting of all zero terms.

10. SQUARING - DWANDWA YOGA

We can use the Urdhava method for squaring. There is another method for squaring using Dwandwa yoga sutra (or the Duplex combination).

To find the Square of the numbers we find the duplexes of the digits.

PROCEDURE :

Level 1 : If we have a one digit number. To Duplex (D) of one digit number we simply square the digit. For example 3, $D = 3^2 = 9$.

Level 2 : For two digit number. We find the duplexes of the digits as double the product of two digits, for ex.

$$35, \quad D = 2\,(3 \times 5) = 2\,(15) = 30$$

Level 3 : For three digit number we find the duplexes as the sum of the square of the middle digit and double the product of the first and third digits as

$$\begin{aligned} 327,\ D &= 2 \times 3 \times 7 + 2^2 \\ &= 42 + 4 = 46 \end{aligned}$$

Level 4 : For four digit number we find the duplexes as the sum of,
double the product of the first and fourth digits, and,
double the product of the second and third digits as

$$\begin{aligned} 1456 &= 2 \times 1 \times 6 + 2 \times 4 \times 5 \\ &= 12 + 40 = 52 \end{aligned}$$

Similarly we can extend it even for the larger digit numbers.

Illustration 19 : Calculate 931^2.

Step 1 : Duplex of 1^{st} digit is

$$1^2 = 1$$

$$\therefore 931^2 = \;/\quad/\quad/\quad/\quad/1$$

Step 2 : Duplex of Ist and 2nd digits i.e. double the product of two digits

We get for 31 ; $2 \times 3 \times 1 = 6$

$\therefore\ 931^2 = \ / \quad / \quad / 6 / 1$

Step 3 : Duplex of Ist , 2nd and 3rd digits is

$931 = 2 \times 9 \times 1 + 3^2 = 18 + 9 = 27$

$\therefore\ 931^2 = \ / \quad / {}_2 7 / 6 / 1$

Step 4 : Duplex of 2nd and 3rd digits; double the product of two digits (level 2).

We have $93 = 2 \times 9 \times 3 = 54$

$\therefore\ 931^2 = \ / \ {}_5 4 / \ {}_2 7 / 6 / 1$

Step 5 : Duplex of 3rd digit only (level 1)

We get $9^2 = 81$

$\therefore\ 931 = 81 / {}_5 4 / {}_2 7 / 6 / 1$

Carry over the excess digits $= 866761$

Hence $931^2 = 866761$

Illustration 20 : $3298^2 = 330\bar{2}$

$= 9 / {}_1 8 / 9 / {}_{\bar{1}} \bar{2} / {}_{\bar{1}} \bar{2} / 0 / 4$

$= 1088\bar{3}\ \bar{2}04 = \underline{\underline{10876804}}$

Illustration 21 : $20289^2 = 203\bar{1}\bar{1}^2$

$= 4 / 0 / {}_1 2 / \bar{4} / 5 / \bar{6} / \bar{5} / 2 / 1 = 412\bar{4}5\bar{6}\bar{5}21$

$= 411643521$

DISCUSSION

Once again we observe the emphasis on mental working. This approach when regularly practised leads to a more agile and alert mind. The results can be obtained by working either from L.H.S. or R.H.S. as convenient.

11. CUBING

11.1 Introduction

In the previous section we have learnt now to square the number easily in one line. Similarly we can do the cubing of the numbers using the sub sutra Anurupyena, which means 'proportionately'. The following sections illustrate the step by step procedure of computing the cube of numbers.

In the general case the first term is cube of the first part (say a^3) and each of the subsequent three terms is obtained by multiplying the first term by the second part (b) and dividing by the first part. That is the subsequent terms are the G.P. of the ratio of 2 digits. Then we double the 2^{nd} and 3^{rd} terms, add and carry.

Illustration 22 : 23^3.

$$\begin{array}{cccccc} 23^3 & = & 8 & 12 & 18 & 27 \\ & & & 24 & 36 & \\ \hline & & 8 & 36 & 54 & 27 \end{array}$$

1^{st} term $= 2^3 = 8$

2^{nd} term $= 8 \times 3/2 = 12$

etc.

$$= 8\;{}_{3}6\;{}_{5}4\;{}_{2}7$$

$$= 12167$$

Illustration 23 : 34^3

$$\begin{array}{cccccc} 34^3 & = & 27 & 36 & 48 & 64 \\ & & & 72 & 96 & \\ \hline & & 27 & 108 & 144 & 64 \end{array}$$

$$= 27\;{}_{10}8\;{}_{14}4\;{}_{6}4 = 39304$$

Cube of a three digit number can be obtained in two steps.

12. STRAIGHT DIVISION

There is yet another method of division in Vedic system. This involves a bit more of mental working but the answer comes straight away in one line. The method is evolved by the combination of two Vedic sutras, namely Urdhava Tiryak and Dhwajanka (meaning on top of the flag).

Illustration 24: Divide 4 6 9 5 6 by 6 5

$$6\;5 \div 4\;6\;9\;5\;6$$

Step 1 : Out of the divisor 65, we put down first digit (i.e. 6) in the divisor column as operator and other digit (i.e. 5) on top of the flag as flag digit as shown below:

$$\begin{array}{l} \nearrow \text{Flag Digit} \\ 5 \,/\, 4\;6\;9\;5\;6 \end{array}$$

(operator) 6

The entire division is to be done by 6.

Step 2 : As one digit has been put on the top we allot one place (at the right end of D) to the remainder portion of the answer and mark off from the other digits by vertical dots

$$\begin{array}{l} \;\;5 : 4\;6\;9\;5 : 6 : \\ 6 \end{array}$$

- Here last digit 6 has been marked off

Step 3 : (i) We divide 46 (of dividend) by 6 and get 7 as the quotient and 4 as remainder, we put down 7 as the first quotient digit and prefix the remainder (4) up before the next digit (i.e. 9) of the dividend. Then our actual gross dividend (G.D.) is 49 for next step.

$$\begin{array}{cccccccc} 6^{5} & : & 4 & 6 & {}_{4}9 & 5 & : & 6 : \\ \hline & & 7 & & & & & \end{array}$$

(ii) from 49 we subtract the product of the flag digit (i.e. 5) and the first quotient digit (i.e. 7). So we get 49 -

$$\begin{array}{r} 49 \\ 7 \times 5 = 35 \\ \hline 14 \\ \hline \end{array}$$

Thus 14 is our net dividend for further operation.

Step 4 : (i) Now divide this net dividend i.e. 14 by 6 we get 2 as second quotient digit and 2 as remainder which is placed before the next digit of dividend (i.e. 5). Now our Gross dividend is 25.

$$\begin{array}{l} 6^5 : 4\ 6\ 9\ 5 : 6 : \\ \quad\quad\ \ 4\ 2 \\ \hline \quad\ 7\ 2 \\ \hline \end{array}$$

(ii) Again from 25 we subtract the product of the flag digit (5) and the 2nd quotient digit (i.e. 2). So we get 25 - (2x5) = 25 - 10 = 15. This is the net dividend.

Step 5 : (i) On dividing 15 by 6 we get 2 as third quotient digit and 3 as remainder to be placed before 6. Now 36 is our next gross dividend

$$\begin{array}{l} 6^5 : 4\ 6\ 9\ 5 : 6 : \\ \quad\quad\ \ 4\ 2\ 3 \\ \hline \quad\quad 7\ 2\ 2 \end{array}$$

(ii) Out of 36 we subtract the product of the flag digit (5) and the 3rd quotient digit (i.e. 2). So we get 36 - (2x5) = 36 - 10 = 26.

Hence 26 is the remainder.

$$\begin{array}{l} 6^5 : 4\ 6\ 9\ 5 : 6 : \\ \quad\quad 4\ 2\ 3 \\ \hline \quad 7\ 2\ 2 : 2\ 6 : \end{array}$$

Therefore Q = 7 2 2

R = 2 6

13. SQUARE ROOTING

To find the square root of the number using the Vedic Sutra is very simple, easy and interesting. The sub sutra named Dwandwa Yoga (द्वन्द्व योग) or Duplex process has two meanings. First one is by squaring and the second one is cross multiplication. This sutra is very easy to apply. Using this sutra we have seen how to find the squares of the numbers. (Item 10).

The step by step process of computing the square root is illustrated in the following examples.

Illustration 25 : Find the Square root of 4489.

Step 1 : First we make the pairs of two digits each. R.H.S. to L.H.S. from the given number, and write as 44 : 89 :

The first pair of two digits from L.H.S. left over or the single digit (if number of digits is odd) is written outside the dots and remaining pairs are written in between the dots.

Step 2 : Now we find the square of the first nine natural numbers as 1, 4, 9, 16, 25, 36, 49, 64, 81 and choose one of them which is closest and lesser than the first pair

(i) In this case square of 6 is suitable. So 6 is the first quotient digit. Which is placed with in the pair of dots. After this step we find the remaining part of the square root using straight division process (Item No. 12). So subtract the square of 6 (i.e. 36) from 44 we get (44 - 36 = 8) 8 as a remainder which is prefixed as the first digit of the next pair (i.e. 8). Hence 88 is the dividend.

(ii) For finding the divisor we double the first quotient digit (i.e. 2 x 6 = 12) obtained in step 2 (i) and write as follows

$$\begin{array}{l|l}
 & 44 : {}_{8}89 : \\
12 & \\
\hline
 & : 6 : \\
\hline
\end{array}$$

So 12 is our divisor. The entire division is done by 12.

Step 3 : Now divide 88 by 12 we get 7 as a quotient and 4 as the remainder which is prefixed to the next digit 9. So 49 is our next gross dividend

$$\begin{array}{l|l}
 & 44 : {}_{8}8\ {}_{4}9 : \\
12 : & \\
\hline
 & : 6 : 7 : \\
\hline
\end{array}$$

Now subtract the square of the quotient digit (i.e. $7^2 = 49$) obtained in (Step 3(1)) from the gross dividend 49, we get $(49 - 49 = 0)$ 0 as quotient and 0 as remainder. Hence the square root is 67. Which is an exact square.

Notes : (1) We have just seen how easy it is to find the square root of any big number.

(2) If a number contains n digits

(i) If n is odd than the square root of a number shall have $\frac{n+1}{2}$ number of digits.

(ii) If n is even than the square root of number shall have $\frac{n}{2}$ digits.

(iii0 If the number is of pure decimal then the number of digits in the square is double that in the square root.

(iv) If the square root is not an exact number, the decimal shall be put after the number of digits equal to the number of pairs formed.

Illustration 26 : Find the square root of 17.

$$\begin{array}{l|l}
 & 17 : 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0 \\
8 : & \quad :1\ 2\ 3\ 2\ 2\ 6\ 7\ 3 \\
\hline
 & : 4 \quad :1\ 2\ 3\ 1\ 0\ 5\ 6
\end{array}$$

DISCUSSIONS

1. In this chapter we have studied the straight, simple and easy Vedic method of finding the square root of any number.

2. The Dwandwa Yoga Sutra used for squaring of numbers has been employed again even for computing the square root.

3. The step by step Vedic method of square rooting supersedes the present methods in many respects.

4. The checking method can be easily and directly applied for checking the corrections of results in case of the exact square roots.

14. SQUARE ROOTS OF POLYNOMIALS

It is interesting to observe that the square root of polynomials can also be computed by following the procedure for determining the square root of a number. The only difference is that there is no carrying. The answer is obtain in just one line (Ref. 8 & 10).

Illustration 27 : Obtain $\sqrt{4 + 12x + 13x^2 + 6x^3 + x^4}$

(As a power series in x).

Solution

Answer :
$$\begin{array}{ll} 4 & 4 + 12x + 13x^2 + 6x^3 + x^4 \\ \cline{2-2} \pm & (2/ + 3x + 1x^2 + 0x^3 + 0x^4) \end{array}$$

check : $(2 + 3x + x^2)^2 = 4 + 12x + 13x^2 + 6x^3 + x^4$

The procedure is the same as for determining the square root of a number, except that there is no carrying. The steps are

1. $\sqrt{4} = 2$

This leads to one solution, and $-\sqrt{4} = -2$ leads to the second. The 2 is written down as the first term in the answer.

2. Doubling this 2, we obtain the divisor, 4. A diagonal storke now separates this 2 from those ensuing terms which contribute to the duplex (viz 3^2, 2.3.1, 2.3.0 + 1^2, 3.0 + 1.0 etc.)

3. The second term in the answer is given by $12 \div 4 = 3$.

4. $13 - 3^2 = 4$ and $4 \div 4 = 1$, which is the third term in the answer.

5. The duplex is now 2.3.1. = 6, and $6 - 6 = 0$, so that

   $0 \div 4 = 0$ (fourth term of the solution)

6. The duplex is $2.3.0 + 1^2 = 1$, and $1 - 1 = 0$, whence

   $0 \div 4 = 0$ (fifth term of the solution)

All subsequent terms are found to be zero, and so the solution is exact. (Ref. 1,8 & 10).

15. COMBINED ARITHMETIC OPERATIONS

With the Vedic system it is possible to perform in one line combined operations of Addition, Multiplication and Subtraction. Even the combined addition and division operations can be performed in one line. Further, there is an element of choice available in every case. For example four different methods are available for computing the series of multiplication and addition in one line (Ref. 8 & 10). Only a couple of examples are presented here.

Illustration 28 : Evaluate 213x426+354x631+652x172. We are going to carry out three "vertically and crosswise" multiplication simultaneously.

$$\begin{array}{lcccccc}
\times & & 2 & 1 & 3 & & \\
 & & 4 & 2 & 6 & & \\
+ & & & & & & \\
 & & 3 & 5 & 4 & & \\
\times & & 6 & 3 & 1 & & \\
+ & & & & & & \\
\times & & 6 & 5 & 2 & & \\
 & & 1 & 7 & 2 & & \\
\hline
 & 4 & 2 & 6 & 2 & 5 & 6 \\
 & & & 10 & 12 & 5 & 2 \\
\hline
 & & & & = & \multicolumn{2}{r}{426256}
\end{array}$$

The larger digits can be avoided by using Vinculum.

Illustration 29 : Divide (2382+5617+7836+6195) by 7. We can record the sum, the working, and the solution as follows.:-

$$\begin{array}{ccccc}
 & \bar{1} & 1 & 3 & \\
\hline
 & 2 & 3 & 2^8 & 4^2 \\
1^5 & & 6 & 1 & 1^7 \\
1^7 & & 1^8 & 1^3 & 1^6 \\
1^6 & & 1 & 1^9 & 1^5 \\
\hline
3 & 1 & 4 & 7 & 1_7 \\
\hline
\end{array}$$

16. SIMULTANEOUS LINEAR EQUATIONS : (Ref. 1,4,8 & 10)

For 3 equations in 3 unknowns we can use the partitioning technique, and compute the 6 cofactors of two portions (written outside) using Urdhava Sutra (see Sec. 17). Further, the four 3 x 3 determinants are computed mentally by using Urdhava, expending by using the appropriate column. In the third step the values of the unknown variables are obtained directly. The whole working is shown below:

Illustration 30 :

$$\begin{array}{r|rr|rcr|r} & xD & yD & zD & & D & \\ -8 & 4x & -2y & +3z & = & 8 & -5 \\ 14 & 2x & -3y & +z & = & 1 & -17 \\ 13 & 3x & +2y & -z & = & 3 & 4 \\ \hline \end{array}$$

$$zD = 8 \times 13 - 14 \times 1 - 8 \times 3 = 66 \text{ etc.}$$

$$\therefore D = 33 \; ; \; xD = 33 \; ; \; -yD = -33 \; ; \; zD = 66$$

Whence $x = y = 1 \; ; \; z = 2$

This method can be further extended to solve for four unknowns. Further, using the pivoting technique (Ref. 8) it is now possible to solve with equal ease and simplicity even eight or ten simultaneous equations. The total working space required is hardly two pages (Ref. 10).

17. INVERSION OF MATRICES : (Ref. 8 & 10)

The Urdhava Sutra can be used to compute the inverse of a 3 x 3 matrix following the well known procedure of writing the inverse of a 2 x 2 matrix. Through mental working process the cofactors are computed and divided by the determinant of A.

18. CURVE FITTING

It is possible to do both mathematical and statistical curve fitting easily by using Vedic Sutras. The procedure used is based on the Urdhava sutra, already explained in item 10 & 16. Further, there are interesting methods based on other sutras, of precise curve fitting. However, only one illustration of fitting a regression line is presented below. (Ref. 8 & 10).

Illustration 31: Fit a regression line of y on x to the four pairs of points given below. The complete working is written down as follows:

y	x
8	5
7	6
4	7
3	9

$$\left.\begin{array}{rcl} 22 & = & 4a + 27b \\ 137 & = & 27a + 191b \end{array}\right\} \text{Normal equations}$$

$$\therefore\ \bar{2}65y = \bar{1}7x + 5\bar{4}$$

ie, $-135y = -3x + 46$ (Regression line of y on x)

The steps are as follows. Under the y-column we write $\Sigma y (=22)$, Σyx $(=137)$, under the x-column we write Σx $(=27)$ and Σx^2 $(=191)$, and between these two totals we put n $(=4)$ and Σx $(=27)$. Finally we solve simultaneously for a and b.

Note that for least squares linear regression of x on y, the roles of x and y are interchanged, and we have :

Regression line of x on y : $x - a' - b'y = 0$

Normal equations: $$\left\{\begin{array}{l} \Sigma x - na' - b'\Sigma y = 0 \\ \Sigma xy - a'\Sigma y - b'\Sigma y^2 = 0 \end{array}\right.$$

19. EVALUATION OF LOGARITHMS AND EXPONENTALS

If we use the Taylor's theorem, it is possible for us to compute the logarithms and exponentials by using the vertically and crosswise sutra. As a first step we take the differential with respect to x and then compare the coefficients. Ref. (8 & 10).

Illustration 32 : Evaluate log (1.2).

The complete working is as follows:

$$\text{Let } \ln(1 + 2x) = \overset{0}{a} + \overset{2}{bx} + \overset{-2}{cx^2} + \overset{8/3}{dx^3} + \overset{-4}{ex^4} + \overset{6.4}{1\,x^5} + \cdots \qquad (1)$$

Differentiating with respect to x,

$$\frac{2}{1 + 2x} = \overset{2}{b} + \overset{-4}{2cx} + \overset{8}{3dx^2} + \overset{-16}{4ex^3} + \overset{32}{5fx^4} + \overset{-64}{6gx^5} + \cdots \qquad (2)$$

$$\therefore \quad \ln\ 1.2 = 0.18232$$

Illustration 33 : Evaluate exp. (.18232).

To keep the figures small, we can use $0.18232 = 0.2\bar{2}232$. The complete working is as follows:

$$\text{Let } \ln(A + Bx + Cx^2 + Dx^3 + \ldots) = 0 + 2x + \bar{2}x^2 + 2x^3 + 3x^4 + 2x^5$$

Differentiate with respect to x:

$$2 \quad 0 \quad \bar{2} \quad 22\tfrac{2}{3} \quad 48$$

$$\therefore \frac{B + 2Cx + 3Dx^2 + 4Ex^3 + 5Fx^4 + 6Gx^5 + \ldots}{A + Bx + Cx^2 + Dx^3 + Ex^4 + Fx^5 + \ldots} = 2 - 4x + 6x^2 + 12x^3 + 10x^4$$

$$1 \quad 2 \quad 0 \quad -\tfrac{2}{3} \quad 5\tfrac{2}{3} \quad 9.6$$

$$\therefore \quad \exp\,(0.18232) = 1.2000$$

20 . CHANGE OF ROOTS OF EQUATIONS

In actual applications frequently we formulate an equation, get its roots, and then study the effect of changing coefficients etc. Vedic sutras provide very direct solution to find how to change a polynomial to obtain another whose roots are related to the original equation by being -

A. Opposite in sign., B. Reciprocals, C. Multiples or sub-multiples

Illustration 34 : To obtain an equation whose roots are opposite in sign to those of a given equation. We transpose the sign of every alternate term, beginning with the second:

1) $x^4 - 2x^3 - 13x^2 + 38x - 24 = 0$ has roots 1, 2, 3, -4.

$x^4 + 2x^3 - 13x^2 - 38x - 24 = 0$ has roots -1, -2, -3, 4.

(2) $2x^2 + 5x - 3 = 0$ has roots $\frac{1}{2}$, -3.

$2x^2 - 5x - 3 = 0$ has roots $-\frac{1}{2}$, 3.

Illustration 35 : To obtain an equation whose roots are reciprocals of those in a given equation. We reverse the order of the coefficients :

(1) $2x^3 + 3x^2 - 8x + 3 = 0$ has roots $\frac{1}{2}$, 1, -3. The coefficients are 2, 3, -8, 3,

$3x^3 - 8x^2 + 3x + 2 = 0$ has roots 2, 1, $-\frac{1}{3}$

(2) $15x^2 - 13x + 2 = 0$ has roots $\frac{2}{3}$, $\frac{1}{5}$.

$2x^2 - 13x + 15 = 0$ has roots $\frac{3}{2}$, 5.

Illustration 36 : To obtain an equation whose roots are all equal to those of a given equation multiplied by a given number.

This comes undere the 'Proportionately formula of Vedic mathematics.

We multiply the coefficients of the given equation by successive powers of the given number, starting with zero. :

$x^2 + 2x - 15 = 0$ has roots 3, -5.

Suppose we want an equation with double those roots, i.e. 6, -10. Multiply the coefficients by $2^0, 2^1, 2^2$:

$x^2 + 2.2x - 2^2.15 = 0$

or $x^2 + 4x - 60 = 0$ is the required equation.

21. COMPUTATION OF TRIGONOMETRICAL FUNCTIONS

By using left to right method of calculations which enables the most significant digit of the answer first then the next most significificent and so on, And with the help of series expansions for the trigonometric functions such as Sin x, Cos x & Tan x etc. these functions can be evaluated for any angle.

For illustration the following examples are considered. Further details are available elsewhere (Ref. 1, 8 & 10).

Illustration 37 : Evaluate cosine 0.2 radians.

Cosine : The series expansion for cos x is :

$$\text{Cos } x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} \quad \text{where } x \text{ is in radians.}$$

If we write this as cos x = 1 + A + B + C + D + ...

We see that $B = \frac{A^2}{6}$, $C = \frac{AB}{15}$, $D = \frac{AC}{28}$, etc.

This means that once we have obtained the value of $\frac{x^2}{2}$ the next term is obtained by squaring this and dividing by 6, the next by multiplying the two previous terms and dividing by 15, and so no.

For Cosine 0.2 radians. We set up a chart as follows :

$1 - x^2/2$		$1.0\ \bar{2}$	row 1
$x^4/24$	6	$.0\ 0\ 0\ 0\ 6\ 6\ 6\ 6\ 6\ 6$	2
$-\ x^6/720$	15	$\bar{1}\ 1_3\,1_3\,1_3\,1_3$	3
cos 0.2	=	$1.0\ \bar{2}\ 0\ 0\ 6\ 6\ 5\ 7\ 7\ 7$	4

21.1 Hyperbolic Functions : These can also be expressed simply in series form, and so can be evaluated like the previous example.

Illustration 38 : Sin h 0.41

We have sin hx $= x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!}$ ....

x		$0\ .\ 4\ 1$
$x^3/3!$	6	$1_2\ 2_{\bar{2}}\ \bar{5}_{\bar{1}}\ \bar{1}_{\bar{2}}\ \bar{3}_{\bar{1}}$
$x^5/5!$	2	$1_0\ 0_1\ \bar{4}_0$
sin h 0.41	=	$0\ .\ 4\ 2\ 2\ \bar{4}\ \bar{1}\ \bar{7}$

Sin h 0.41 = 0.42158 to 5 decimal places

Similarly we can evaluate cos hx and tan $h^{-1}x$ using :

$$\cos hx = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \ldots\ldots$$

$$\tan h^{-1}x = x + \frac{x^3}{3} + \frac{x^5}{5} + \frac{x^7}{7} + \ldots\ldots$$

21.2 Inverse of Sine, Cosine and Tangent : The ability to make calculations from left to right means that we can obtain the most significant digit of an answer first, then the next most significant and so on. In this chapter and the next we show how the digits of an answer, can be used to obtain the next digit.

INVERSE OF SINE :

Illustration 39 : Find $\sin^{-1} 0.4$ to 4 D.P.

$$\text{Since } \sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} \;\ldots\ldots\ldots$$

$$x = \sin x + \frac{x^3}{3!} + \frac{x^5}{5!} + \ldots\ldots\ldots$$

Here we are given sin x = 0.4, and we have to find x

$$x = 0.4 + \frac{x^3}{3!} - \frac{x^5}{5!} + \ldots\ldots$$

We therefore set up a chart to evaluate the terms on the RHS of this equation :

sin x		$0\,.\,4$	row 1
$x^3/3!$	6	$1_2\;1_4\;6_{\bar{1}}\;1_2$	2
$-x^5/5!$	2	$\bar{1}\;\;0_{\bar{1}}$	3
x =		$0\,.\,4\;1\;\;1\;\;5\;\;1$	4
$(x^2$ =		$.2_{\bar{4}}\;\bar{3}_{\bar{2}}\;\bar{1}_{\bar{1}}\;\;3_2\,)$	5

The answer appears in row 4, and rows 2, 3 and 5 are formed digit by digit as x is found. We therefore work vertically and as each digit of x is found it is used to obtain the next one. (ref. 1, 8 & 10).

22. TRANSCENDENTAL EQUATIONS

The method described in the computations of trigonometric functions can be applied to the solution of transcendental equations.

Illustration 40 : x + sin x = 1

$$x + x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots\ldots\ldots = 1$$

$$x = 0.5 + \frac{x^3}{12} - \frac{x^5}{240} + \frac{x^7}{2.7\,!} \;\ldots\ldots$$

0.5		$0\,.\,5$
$x^3/12$	12	$1_3\;1_1\;1_2\;2_2$
$-x^5/240$	2	$\bar{1}_1\;\bar{5}_{\bar{1}}$
x =		$0\,.\,5\;\;1\;\;1\;\;0\;\;3$
$(x^2$ =		$.3_{\bar{5}}\;\;\bar{4}_0\;1\;\;1_2\;\;\bar{1}_1)$

The method is quick and efficient if x is small, and can be extended to any number of decimal places.

Kepler's Equation : This is an equation of great importance in positional astronomy. The equation is $M = E - e \sin E$

where M, E are the mean and eccentric anomalies respectively of a planet in its orbit, and e is the eccentricity of the orbit.

We will take $M = 0.30303$ radians and $e = 0.016722$ - which is the eccentricity of the Earth's orbit. We want to find E.

Illustration 41 : $0.30303 = E - 0.16722 \sin E$

$$E = 0.30303 + 0.016722 \left(E - \frac{E^3}{3!} + \frac{E^5}{5!} - \ldots\ldots\ldots\ldots\right.$$

M		$0\,.\,3\;0\;3\;0\;3\;0\;0$	1
eE		$1_{\bar{4}}\;\bar{5}_3\;2_{\bar{6}}\;\bar{5}_1\;2_1\;1_{\bar{3}}$	2
$-eE^3/3!$	6	$\bar{1}_{\bar{1}}\;2_{\bar{1}}\;\bar{1}_3\;\bar{5}_0$	3
$eE^5/5!$	2	4_0	4
E	=	$0\,.\,3\;1\;\bar{2}\;1\;0\;1\;0$	5
$(E^2$	=	$.\;1_{\bar{1}}\;0_{\bar{4}}\;\bar{5}_{\bar{1}}\;\bar{1}_2\;3_{\bar{4}}$	

where $e = 0.16722 = 0.02\bar{3}\bar{3}22$

Bring 0.3 into the answer.

Row 2 : We multiply e and E : $CP\left[\begin{smallmatrix}2\\3\end{smallmatrix}\right] = 6 = 1_{\bar{4}}$

Bring 1 into the answer.

Row 2 : $CP\left[\begin{smallmatrix}2\bar{3}\\31\end{smallmatrix}\right] = \bar{7}, \bar{7} + \bar{4}\bar{0} = \bar{5}_3$

Bring down $\bar{2}$ into the answer etc. For further details refer (Ref. 1,8 & 10).

23. FUNCTIONS OF POLYNOMIALS

A well known method of obtaining a power series for any function makes direct use of Taylor's series or Maclau rin's series which is the conventional method. The drawback is that higherorder derivative tend to become increasingly difficult to evaluate. Therefore an approach free from such a disadvantage is more apealing.

Illustration 42 : By using the algebraic equivalent of VERTICALLY AND CROSSWISE the complete working and solution is given for following example. For working details refer (8).

(1) $(2 + 3x + 5x^2 + 3x^3) \div (2 - x + 3x^2)$

$$\begin{array}{ll|l} & 2^{-1} \quad 3 & 2 + 3x + 5x^2 + 3x^3 \\ \hline \text{Solution} & & 1 + 2x + 2x^2 - \frac{1}{2}x^3 - 3\frac{1}{4}x^4 - \frac{7}{8}x^5 + \frac{5}{48}x^6 + \ldots\ldots\ldots\ldots \end{array}$$

(2) Evaluate $\dfrac{1}{1 - x - x^2}$

Working and Solution :

$$\begin{array}{l|l} -1 \quad -1 & 1 \\ \hline \text{Solution} & 1 + 1x + 2x^2 + 3x^3 + 5x^4 + 8x^5 + 13x^6 + 21x^7 + \ldots\ldots \end{array}$$

(3) Find $\sqrt[3]{x^6 + 12x^5 + 57x^4 + 136x^3 + 171x^2 + 108x + 27}$ obtaining the solution as a descending power series in x.

The complete working and solution is written down as follows :

$$\begin{array}{ll|l} & 3 & x^6 + 12x^5 + 57x^4 + 136x^3 + 171x^2 + 108x + 27 \\ \hline \text{Solution} & & 1x^2/+4x + 3 + \frac{0}{x} + \frac{0}{x^2} + \frac{0}{x^3} + \ldots\ldots \end{array}$$

(4) Expand $(1 + 3x - 2x^2)^{10}$

Let $\quad (1 + 3x - 2x^2)^{10} = a + bx + cx^2 + dx^3 + ...$

Take logs, differentiate with respect to x and equate coefficients using Vedic method to get the solution straight away. This procedure can also be used to find the fractional powers of the polynomials.

Further, it is possible even to compute the imaginary exponents also by working on the same lines (Ref. 8). Once again the Urdhava Sutra is actively used.

24. FUNCTIONS OF BIPOLYNOMIALS

Just as the facility for expressing functions of polynomials as power series opens up ways of handling function of a single variable using Urdhava Sutra. A similar facility opens up possibilities with functions of two variables.

Illustration 43 : Divide

$$\left(\begin{array}{llll} 2 & + 4x & + 0x^2 & + 3x^3 \\ +3y & + xy & + 2xy^2 & \\ + y^2 & + 3xy^2 & & \end{array}\right) \quad \text{by} \quad \left(\begin{array}{ll} 1 & + 3x \\ +2y + & xy \end{array}\right)$$

The complete working is written down as follows, with the solution appearing at bottom right :

$$\begin{array}{ccc|cccc|c} 1 & + & 3x & 2 & + 4x & + 0x^2 & + 3x^3 & \\ +2y & + & xy & +3y & + xy & + 2x^2y & & P \\ & & & +y^2 & + 3xy & & & \\ \hline & & -3 & 2 & -2x & 6x^2 & -15x^3 & \\ -2 & & -1 & -1y & 6xy & -26x^2y & 102x^3y & S \\ & & & 3y^2 & - 17xy^2 & -14x^2y^2 & ...... & \end{array}$$

This method is applicable when the constant term in the divisor is unity. There is no divison to be performed, only addition and multiplication (and subtraction, when adding negative terms.

25. NON LINEAR DIFFERENTIAL EQUATIONS (Ref. 8, 10)

The method of series solution of differential equations is well known. This when combined with the Vedic Mathematical System consisting of sixteen sutras as re-structured by Shri Bharati Krishen Tirth ji makes this an immensely powerful approach. This procedure can be used to handle a vide range of problems particularly those where the boundary conditions are given at the origin or at a single point.

Illustration Solve $2y + (y')^2 = 13 + 18x + 6x^2$,

Where $y(1) = 6$

The boundary conditions not being given at the origin, we have a choice between expanding y as a power series in $(x - 1)$, or changing to a new independent variable $z = x - 1$. Both approaches amount to the same thing.

Following the former course, we have:

$$2y + (y')^2 = 37 + 30(x - 1) + 6(x - 1)^2,$$

where $y(1) = 6$

The solution now proceeds:

$$y = \overset{6}{A} + \overset{5}{B}(x-1) + \overset{1}{C}(x-1)^2 + \overset{0}{D}(x-1)^3 + \overset{0}{E}(x-1)^4 + F(x-1)^5 + \ldots\ldots\ldots$$

$$y' = \underset{5}{B} + \underset{2}{2C}(x-1) + \underset{0}{3D}(x-1)^2 + \underset{0}{4E}(x-1)^3 + 5F(x-1)^4 + 6G(x-1)^6 + \ldots\ldots$$

We first find $A = 6$, then $B^2 = 25$

This leads to two solutions, one with $B = +5$ (given above), and one with $B = -5$.

∴ i.e. the two solutions are:

$$\underline{y = 6 + 5(x-1) + (x-1)^2} \qquad \text{(exact solution)}$$

$$\& \qquad \underline{y = 6 - 5(x-1) - 2(x-1)^2 - 0.2(x-1)^3 - 0.11(x-1)^4 \ldots\ldots}$$

Illustration Solve $y + 2y' y'' = \exp(x) \cos 2x \ldots\ldots$ (1)

where $y(0) = 1$, and $y'(0) = 1, 1, y'''(0) = 0$

A power series expansion for exp(x) cos 2x can be obtained from MacLaurin's Theorem. Alternatively, as shown below, write down the expansion for exp(x), and below it write the expansion for cos 2x, keeping like powers in the same column:

$$y + 2y' y'' = (1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \frac{x^6}{6!} + \ldots\ldots) \text{ times}$$

$$(1 + 0 - \frac{4x^2}{2!} + 0 + \frac{16x^4}{4!} + 0 - \frac{64x^6}{6!} + \ldots\ldots)$$

The right hand side coefficients of the successive powers of x can now be worked out as needed, using the "vertically and cross-wise" procedure.

i.e. the solution is,

$$\underline{y = 1 + x - \frac{1}{8}x^4 - \frac{11}{240}x^5 - \frac{7}{840}x^6 \ldots\ldots}$$

26. INTEGRO DIFFERENTIAL EQUATIONS (Ref. 8)

Illustration Solve $y^2 + 2(\frac{dy}{dx})^2 + y \int_0^x y\,dx = 4 + 4x + 60x^2 - 8x^3 + 9x^4 + 3x^5$

where $y(0) = 2$, $y'(0) = 0$

Assume

$$y = \overset{2}{a} + \overset{0}{b}x + cx^2 + dx^3 + ex^4 + fx^5 + gx^6 + \ldots$$

$$\therefore \quad y' = \overset{0}{b} + 2cx + 3dx^2 + 4ex^3 + 5fx^4 + 6gx^5 + 7hx^7 + \ldots.$$

$$\& \quad \int_0^x y\,dx = 0 + \overset{2}{2}x + \frac{b}{2}x^2 + \frac{c}{3}x^3 + \frac{d}{4}x^4 + \frac{e}{5}x^5 + \frac{f}{6}x^6 + \ldots.$$

Boundary conditions lead to a = 2 and b = 0, as shown above. Succeeding steps involve the solution of simple equations

& $y = 2 - 3x^2$ is one solution.

Further the solution of transcendental equations and partial differential equations have also been obtained (Ref. 8).

27. VEDIC NUMERICAL CODE

It is a matter of interest to note that Vedic scholar of ancient time did not use figures (for big numbers) in their numerical notations but preferred to use Sanskrit alphabet to represent various numbers. This was perhaps done to facilitate the mathematical knowledge stored and transferred on to subsequent generations by way of language, understood to mankind. Thus preservation of their knowledge was better managed and understood by subsequent followers. Sutras and Sanskrit verse are in fact treasures of such kind which can be easily decoded and assimilated even to the modern world.

In this context, it is interesting to point out the honour being given to the Sanskrit language by the American Association for Artificial Intelligence. The full cover page of the AI magazine (volume 6 No. 1 spring 85), an official publication of the American Association for the artificial intelligence is decorated with Sanskrit shlokas. This particular issue also contains a research article of 10 pages by Dr. Rick Briggs, who is a computer scientist at NASA, American Space Research Centre, CALIFORNIA, on, "Knowledge representation in Sanskrit and Artificial Intelligence "

28. THE CODE

The code system in Hindi could be written down as :

क	ट	प	-1	य
ख	ठ	फ	2	र
ग	ड	ब	3	ल
घ	ढ	भ	4	व
ङ	ण	म	5	श
च	त		6	ष
छ	थ		7	स
ज	द		8	ह
झ	ध		9	
		क्ष	0	

Note : Further न and ञ are also be coded as 5

The whole code is contained in verse
कादिनव टादिनव पादिपचंचक याद्यष्टक क्ष शून्यम् च।
This code system can also be written in English as follows :

(i) ka, tta, pa and ya all denotes 1,
(ii) kha, dah, pha, ra all denotes 2,
(iii) ga, da, ba, la all denotes 3,
(iv) gha, dha, bha, va stands 4,
(v) gna, na, ma and sha denotes 5,
(vi) ca, ta and ksha denotes 6,
(vii) cha, tha, sa denotes 7,
(viii) ja, da and ha stands 8,
(ix) jha, dah denotes 9,
(x) Ksa (or Ksudra or shunyam) for 0

The vowels are ignored and in case of conjunct consonants the last consonant is alone counted.

29. **Illustration** : Thus pa pa is 11, ma ma is 55, ta ta is 11, ma ry is 52 and so on.

It was left to the author to select proper consonant or vowel. This gave lattitude to the writers to frame the language of their writings to acquire more than one meanings. For instance, kapa, tapa, papa and yapa all mean 11.

Applications : We now take three illustrations which when decoded provide interesting mathematical insight :

(i) केवलैः सप्तकं गुण्यात्।

Here Saptakam means 7 and Kevalaih represents 143. We are told that in case of 7 out multiplicand should be 143.

Applying Ekanyana corollary of Nikhilam all this means as for 7 (as denominator) multiply 143 by 999 (Refer item 5). This is the answer for -

$$\frac{1}{7} = .\dot{1}\ 4\ 2\ 8\ 5\ \dot{7}$$

(ii) कंसे क्षामदाहखलैर्मलैः

Here Kanse means 17 ksama-daha-khalairmalaih means 0 5 8 8 2 3 5 3.

Thus $\frac{1}{17}$ is obtained by 0 5 8 8 2 3 5 3 x 9 9 9 9 9 9 9 9

i.e. $\frac{1}{17}$ = .0 5 8 8 2 3 5 2 9 4 1 1 7 6 4 7

(iii) Lastly a Shloka for obtaining value of $\pi/10$.

गोपी भाग्यमधुव्रात शृङ्गिशोदधि सन्धिग
खल जीवित खाताव गलहाला रसंधर ॥

This Shloka has two sets of literal meaning, first in praise of Lord Krishna and second for the Lord Shiva. Third meaning (after decoding) gives value of $\pi/10$ to 32 places of decimals as below -

$$\frac{\pi}{10} = .31415926535897932384626433832792 \text{-------}$$

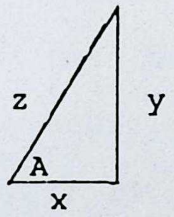
Further, research is revealing very interesting results. For instance the Sacred Gaytri Mantra, when decoded yields the sacred 108 number. Further details are presented in Ref. 10.

Observations : Dr. V.P. Dalal (of Heidelburg University Germany) after knowing above Shloka's merit commented as follows -

It shows how deeply the ancient Indian Mathematicians penetrated, in the subtlety of their calculations, even when the Greeks had no numerals above 1000 and their multiplications were so very complex, which they performed with the help of the counting frame by adding so many times the multiplier; 7x5 could be done by adding 7 on the counting frame 5 times. etc.

30. TRIBHUJANK

The ancient Vedic rishis had complete knowledge and understanding of geometry also. This is very well accepted by all modern scholars. Here in we introduce ourselves to a very interesting, useful and thought provoking facet of ancient knowledge.A set of three numbers x, y and z which satisfy the condition $x^2 + y^2 = z^2$ is called Tribhujank (त्रिभुजांक). This word consists of three parts, Tri (Pronounced as Tree) means three; bhuja means the side; ank means the numbers. Therefore Tribhujank stands for a set of three numbers which essentially represents the three sides of a right angled triangle, having x as the base, y as the height and z as hypotenuse. Tribhujank written in this sequence essentially represents the angle (A) between the sides x and z.



The so called Pythagorus theorem was extensively used by the ancient Vedic people a few thousand years before the Pythagorus period.The following four formulae from Katyayan's sulbha sutra clearly demonstrate their perfect understanding of this simple and important facet of geometry.

1. सम चतुरस्त्रस्य अक्षण्या रज्जू: द्विकरणी।
2. द्वि प्रमाणा चतु: करणी, त्रि: प्रमाणा नवकरणी, चतु: प्रमाणा पोडस करणी।
3. अर्धप्रमाणेन पादप्रमाणम् विधीयते।
4. तृतीयेन नवमोम्सः।

meaning thereby

1. The rope measuring the diagonal of a square can help in constructing the square twice in area as the first.
2. With the help of a rope twice the length of a side of a square, you can construct another of four times in area, thrice in length - nine times and four times in length- sixteen times.
3. Half the length of rope will yield quarter area.
4. One third length of rope will yield one ninth area.

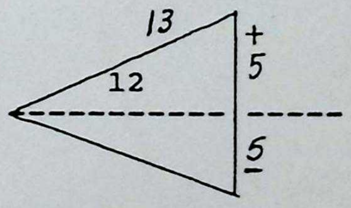
Even one old Babylonian tablet (Plimpton collection of Columbia University, New York) which dates from 1900- 1600 B.C. presents a set of fifteen Tribhujanks decreasing in steps of 1^{o}.

The set of numbers 3, 4, 5 represent one Tribhujank as $3^2 + 4^2 = 5^2$. If we know the hypotenuse and any of the other two sides, the Vedic sutra (संकलन व्यकलनाभ्याम्) Sankalana Vyavakalanabhyam (No. 7) can be used to compute the third side very conveniently. The sutra means by addition and subtraction.

Example 44: If 13 is the hypotenuse (z) and 12 is the base (x) of a Tribhujank and the third side, height is y than

$$y^2 = z^2 - x^2$$

$$= (z+x)\ (z-x)$$



Therefore $y^2 = (13+12)\ (13-12)$

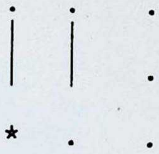
$$y^2 = 25$$

and $y = 5$. In general y can also be -5. (See Fig. 1.)

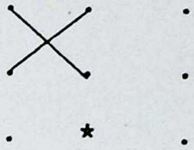
ADDITION AND SUBTRACTION OF TRIBHUJANK

Both of these operations can be easily done by direct application of the Urdhava sutra (No.3) which simply means

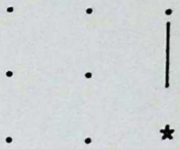
vertically and crosswise;only the specific procedural details have to be noted.The three elements of the new Tribhujank are obtained as follows:



1st element



2nd element



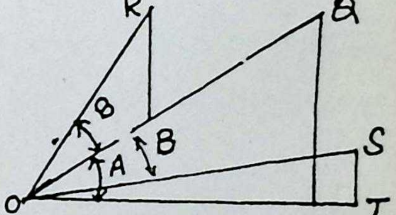
3rd element

If (A) x1, y1, z1 and

(B) x2, y2, z2 are two Tribhujanks containing angles A & B respectively. Than the Tribhujank containing angles A - B and A + B are

A	x1	y1	z1
B	x2	y2	z2
A - B	(x1 x2+y1 y2)	(x2y1-x1y2)	(z1z2)
A + B	(x1x2-y1y2)	(x2y1+x1y2)	(z1z2)

Fig.2



∠ROT =A+B; ∠SOT =A-B

Let us consider one example.

Example 45: Find Tribhujank of angle A- B and A+ B. If

A	4	3	5
B	12	5	13
A - B	(4x12+3x5)	(12x3-4x5)	(5x13)
	63	16	65
Similarly;A+B	(4x12-3x5)	(12x3+4x5)	(5x13)
	33	56	65

Refer Fig. 2.

Example 46: Find the Tribhujank for A-B, where (A) 12, 5, 13 (b) 5, 12, 13

A	12	5	13
B	5	12	13
A-B	(60+60)	(25-144)	169
	120	- 119	169

The negative sign of the second element indicates that the resulting angle is negative.(See fig.7).

We can easily see that the Tribhujank of any angle (-A) can be obtained by simple change of the sign of middle element.

Example 47 : The Tribhujank for 45°, 30°, 60° etc. can be easily found from half of the square and half of an equilateral triangle.(See Figs. 8 & 9).

45°	1	1	$\sqrt{2}$
30°	$\sqrt{3}$	1	2
60°	1	$\sqrt{3}$	2

HALF ANGLE TRIBHUJANK

If (A) x, y, z then A/2 Tribhujank will have the same middle element (y). However the first element shall be the addition of the first and third element (x+z) of A. Further,the third element can be computed by using the other two elements.

Find the half angle Tribhujank of (A) 3, 4, 5.(Fig. 10).

If A) 3, 4, 5 and B) 5, 12, 13

Example 48 : Evaluate (i) $\cot^{-1}(2)+\cot^{-1}(3)$,

(ii) $2\tan^{-1}(3)+\cot^{-1}(2)$

(iii) $\sin^{-1}(3/5)+\tan^{-1}(2)$

We are required to add the angles represented by the individual inverse functions.

(i)

A	2	1	5
B	3	1	$\sqrt{10}$
A + B	5	5	-
	1	1	$\sqrt{2}$

Therefore A + B = 45°

(ii)

A	1	3	$\sqrt{10}$
2A	-8	6	10
B	2	1	$\sqrt{5}$
2A + B	-22	4	-
	-11	2	-

Therefore $2\tan^{-1}(3)+\cot^{-1}(2) = \tan^{-1}(-2/11)$

(iii)

A	4	3	5
B	1	2	$\sqrt{5}$
A + B	-2	11	$5\sqrt{5}$

Therefore $\sin^{-1}(3/5)+\tan^{-1}(2) = \tan^{-1}(-11/2)$

Further Illustrations

Example 49: Evaluate Tribhujank for 120°, 240°, 675°, 105, -15°,

Example 50 : Prove the following :

(i) $$\frac{\sin A + \sin 2A}{\cos A - \cos 2A} = \frac{\cot A/2}{2}$$

(ii) $$\frac{\cos 2B - \cos 2A}{\sin 2B + \sin 2A} = \tan(A-B)$$

(iii) $$\sin 105^{o} + \cos 105^{o} = \cos 45^{o}$$

Let Tribhujank for (A) be x y 1

A	x	y	1
2A	$x^2 - y^2$	$2xy$	1
A/2	$x+1$	y	$\sqrt{(x+1)^2 + y^2}$

Similarly if (B) x1, y1. 1

A	x	y	1
B	$x1$	$y1$	1
A - B	$(x.x1+y.y1)$	$(x1.y-x.y1)$	1
2B	$(x1^2 - y1^2)$	$2x1y1$	1

(i) $$\frac{\sin A + \sin 2A}{\cos A - \cos 2A} = \frac{y + 2xy}{x - (x^2 - y^2)}$$

$$= \frac{y(1 + 2x)}{x + 1 - 2x^2} \left[\text{as } y^2 = 1 - x^2 \right]$$

$$= \frac{y}{1 - x} * \frac{1 + x}{1 + x}$$

$$= \frac{x + 1}{y} = \cot(A/2)$$

(ii) $$\frac{\cos 2B - \cos 2A}{\sin 2B + \sin 2A} = \tan(A-B)$$

$$\frac{x1^2 - y1^2 - x^2 + y^2}{2x1.y1 + 2xy} = \frac{x1.y - x.y1}{x.x1 + y.y1}$$

$$\frac{x1^2 - x^2}{x1.y1 + xy} = \frac{y.x1 - x.y1}{x.x1 + y.y1} \quad \left[\text{as } y^2 = 1 - x^2\right]$$

By crossmultiplying we get,

$$x1^2 + y1^2 = x^2 + y^2 = 1 \quad \text{(Okay)}$$

(iii) $$\sin 105^\circ + \cos 105^\circ = \cos 45^\circ$$

The Tribhujank of 105° is $1-\sqrt{3}$, $1+\sqrt{3}$, $2\sqrt{2}$ (calculated in the previous question 12).

Therefore

$$\frac{1+\sqrt{3}}{2\sqrt{2}} + \frac{1-\sqrt{3}}{2\sqrt{2}} = \frac{1}{\sqrt{2}} = \cos 45^\circ$$

Proved.

Present Method :

$$\begin{aligned}
\sin 105^\circ + \cos 105^\circ &= \sin(180^\circ - 105) - \cos(180^\circ - 105^\circ) \\
&= \sin 75^\circ - \cos 75^\circ \\
&= \sin 75^\circ - \sin(90^\circ - 75^\circ) \\
&= \sin 75^\circ - \sin 15^\circ \\
&= 2\cos 45^\circ \sin 30^\circ \\
&= 2 \cos 45^\circ * \frac{1}{2}
\end{aligned}$$

* (a series of formulae have been used)

$$= \cos 45^\circ$$

proved.

a = angle BOC, b = angle AOC, c = angle AOB

By angle A we mean the angle between the tangents to the arcs AB and AC at A or the angle between the planes AOB and AOC.(See Fig. 1)

31. **Tribhujank for Spherical Triangles**

The six angles a, b,c, A,B,C can each be described by a Tribhujank and three of these Tribhujanks are sufficient to define a particular spherical triangle.

a	4	3	5	A	.	.	.
b	5	12	13	B	.	.	.
c	.	.	.	C	8	15	17

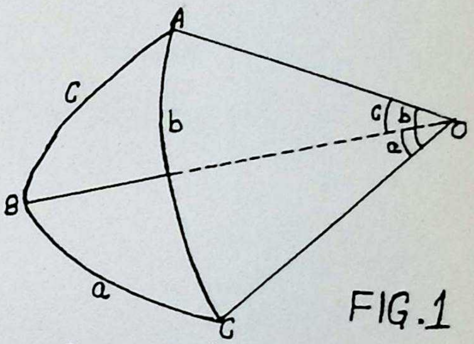
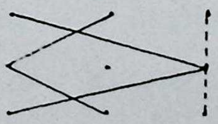


FIG.1

Further, we know that two elements of a Tribhujanks are sufficient to define it. We have a set of four formulae which are somewhat complex and are not easy to remember or apply but when we use the Tribhujank approach very simple patterns immerged which can be used to solve the spherical triangles much more easily.

Example 51: If Tribhujanks of a and b are 3, 4, 5 and 5, 12, 13 respectively and angle C is 7, 24, 25.Find c.

Solution: Using cosine formula, this formula is applied when we know two sides and the angle between them.(See Fig. 2) To find the side opposite to the given angle the procedure is given below-



let the Tribhujank of c be x, y, r then by using the above explained

procedure we have

a	3	4	5
C	7	24	25
b	5	12	13

so, x:r = 3*25*5 + 4*7*12: 5*25*13

= 711 : 1625

Hence (c) 711, -, 1625

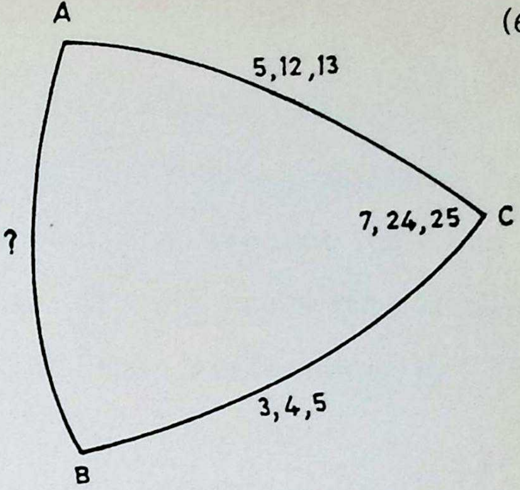


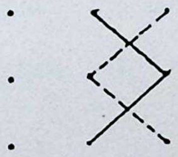
FIG.2

Example 52: Find an angle B between the two sides b & c, whose Tribhujanks are 4,3,5 and 12, 5, 13 and the angle (C) 15, 8,17 opposite to angle C.

Solution : Here

b	4	3	5
c	12	5	13
C	15	8	17

let the Tribhujank of B be x, y, r. By using the sine formulae



we get y:r = 13*3*8: 5*5*17

∴ (B) -, 312, 425

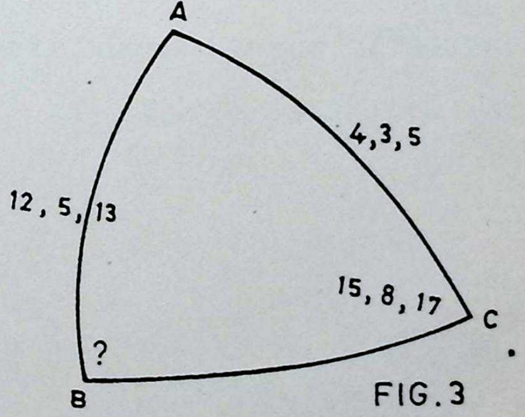


FIG.3

Example 53 : Find the angle B by using the Polar Cosine rule if Tribhujanks of b and c are 12, 5, 13 and 8, 15, 17 and the angle (A) 3, 4, 5.

32. CO-ORDINATE GEOMETRY

We need the equation of a circle with centre (2,3) and tangential to line y = 3x.

We need the radius of the circle, AC, since the circle is tangential to the line $y = 3x$, the sum of the angles between the centre and the x-axis (E) and the centre and the tangent (F) has a gradient of 3. Therefore the second angle (E) and hence the radius of the circle can be obtained by finding the difference of the Tribhujanks (E + F) and E. (See Fig. 1).

Tangential line (E + F)	1	3	$\sqrt{10}$
Centre (E)	2	3	$\sqrt{13}$
(E + F) -E	11	3	$\sqrt{130}$

OR $11/\sqrt{10}$, $3/\sqrt{10}$, $\sqrt{13}$

Since OC $=\sqrt{13}$ $\therefore$ AC $= 3/\sqrt{10}$

Therefore the equation is

$$(x-2)^2 + (y-3)^2 = 9/10$$

Since we are interested in the actual lengths therefore the Tribhujank has to be divided by suitable number (here $\sqrt{10}$).Further, frequently we do not require all the three elements of the Tribhujank.

LENGTH OF PERPENDICULAR

Example 54: What is the length of perpendicular from the point (4,3) to the line $y = 2x$.

Solution: If we subtract the Tribhujank for point from the Tribhujank for the line we shall get the Tribhujank for the remaining angle.The middle element of which shall be the desired length of perpendicular (See Fig. 2).

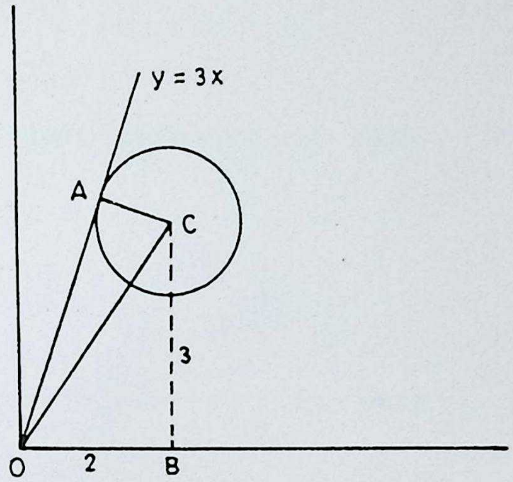


FIG.1

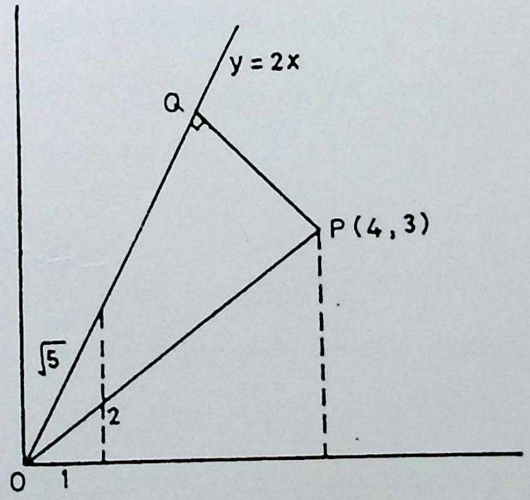

y = 2x
Q
P(4,3)
√5
2
O
1

FIG.2

line	1	2	$\sqrt{5}$
point	4	3	5
Subtracting:	10	5	$5\sqrt{5}$

Triangle OPQ =$10/\sqrt{5}$ $5/\sqrt{5}$ 5

Therefore PQ = $\sqrt{5}$

Example 55: Compute the distance of the point (-2,-3) from the line $4y + 3x = 0$.

line	4	-3	5
point	-2	-3	-
Subtracting	-	18	-

Therefore PQ = 18/5 = 3.6

Example 56: Find the distance of p (5,4) from the line $3y + 4x = 7$.

Solution : This line does not pass through the origin, therefore we should first transpose the origin to any convenient point which lies on the line. We observe that point (1,1) lies on the line therefore transposing the origin to this point we have to find the distance of (4,3) from $3y + 4x = 0$.

line	3	-4	5
point	4	3	-
Subtracting	-	-25	-

Therefore PQ = -25/5 = -5 = 5 (we ignore the sign).

Transformations in a plane :

Example 57 : Rotate P(4,3) through an angle of 90° anticlockwise by using origin as the centre of rotation.

Tribhujank for P(4,3) and 90°.

P	4	3	5
90°	0	1	1
Adding	-3	4	5

Therefore the rotated point is P′ (-3,4).

Example 58 : Rotate P(-3,5) 90° clockwise about the point (2,7).

We transpose the origin to the point of rotation (2,7). The new coordinates of P are (-3 -2, 5-7) =(-5,-2). Now we rotate clockwise this point about(0,0) by 90°.

p	-5	-2	-
90°	0	1	-
Subtracting	-2	5	-

Therefore the rotated point p′(-2,5) relative to original origin is P′(-2+2,5+7) = (0,12).

Example 59 : Rotate the point P(4,3) through an angle (A) 3,4,5 about origin in the anticlockwise direction.

Solution:

P	4	3	5
A	3	4	5
Adding	0	25	25

33. CALCULAS:

We use the Urdhva sutra.

Example 60: $D = x^{2} . e^{3x} . \sin x$

We write the first and second column of three terms.The second column shall contain the first derivative of each term.

$$\begin{array}{ll} x^{2} & 2x \\ e^{3x} & 3e^{3x} \\ \sin x & \cos x \end{array}$$

By using the first two steps of multiplication by the Urdhava method from left to right.

$$D = x^{2} .e^{3x} \sin x \quad \text{(vetically)}$$

$$D_{1} = x^{2} e^{3x} \cos x + x^{2} \sin x \, 3e^{3x} + 2x\, e^{3x} \sin x \quad \text{(crosswise)}$$

$$= x^{2} e^{3x} \cos x + 3x^{2} \sin x \, e^{3x} + 2x\, e^{3x} \sin x$$

The result is obtained directly in a single step and no unnecessary working is required to be done.

Second Derivative

If we are interested to further compute the second derivative of this function consisting of three parts.

The three columns are written as before.

$$\begin{array}{lll} x^{2} & 2x & 2 \\ e^{3x} & 3e^{3x} & 9e^{3x} \\ \sin x & \cos x & -\sin x \end{array}$$

$$D_2 = -x^2 e^{3x} \sin x + 9x^2 \sin x\, e^{3x} + 2e^{3x} \sin x + 2*x^2\, 3e^{3x} \cos x$$

$$+ 2*2x\, e^{3x} \cos x + 2*2x \,.\, 3e^{3x} \sin x$$

We double the terms of vertical simplification of the middle column pairs as before.

Finally:

$$D_2 = 8x^2 e^{3x} \sin x + 6x^2 e^{3x} \cos x + 4xe^{3x} \cos x + 12xe^{3x} \sin x + 2e^{3x} \sin x$$

is the 2nd derivative of the function consisting of the product of the 3 terms, obtained directly in one line working. As per present system of working by parts we shall require more than one page of working. Further this method can be extended to compute the higher derivatives by the direct application of the Urdhava sutra.

Differentiation of a function consisting of ratio of two variables

We know that $\frac{d}{dx}\left(\frac{u}{v}\right) = \dfrac{u\frac{dv}{dx} - v\frac{du}{dx}}{v^2}$

It is interesting to observe that the differential of a ratio of two polynomials can be directly obtained in one line by the simple application of the Urdhava sutra (No.3). The only procedural difference is that we find the difference of the cross product of the coefficients of different powers of x. This is quite similar to the procedure used in solving the set of simultaneous equation (Refer 5, 6 & 7).

Example 61: Differentiate $z = \dfrac{3 + 7x}{1 + 3x}$

$$\frac{dz}{dx} = \frac{(1*7 - 3*3)(1 - 0)}{(1 + 3x)^2} = \frac{-2}{(1 + 3x)^2}$$

where the second bracket contains the difference of the powers of x in the two terms being cross multiplied. Further the vertical multiplication does not appear in the numerator as the difference of the powers of x would be zero.

Example 62: Differentiate $z = \dfrac{2 + 3x + 5x^2}{3 + 4x + 7x^2}$

$$\frac{dz}{dx} = \frac{(3*3-2*4)(1-0) + (3*5-7*2)(2-0)x + (4*5-3*7)(2-1)x^2}{(3 + 4x + 7x^2)^2}$$

$$= \frac{1 + 2x - x^2}{(3 + 4x + 7x^2)^2}$$

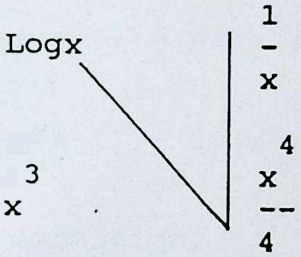
The powers of x in different terms are equal to one less than the sum of the powers of different terms being cross multiplied. This is in accordance with the Ekanyunena sutra (No.14) used f differentiation. Similarly, it is possible to find out the differential of ratio of even bigger polynomials.

Example 63: Differentiate $z = \dfrac{1 + 4x + 2x^2 + 7x^3}{2 + 3x + 5x^2 + 11x^3}$

As already explained the numerator is obtained by using the Urdhava sutra.

Integration of the product of two functions : For this, we first write the function which can be easily differentiated and write the differential of the same.The second function is writen in the second line and write its integration after it, and then apply the Urdhava sutra.The signs of the products are positive and negative alternatively.

Example 64: $I = \int x^3 \log x \, dx$



Therefore,

$$I = \frac{x^4}{4} \log x - \int \frac{1}{x} \quad \frac{x^4}{4} \, dx$$

This can be further integrated by using the Ekadhiken sutra (No.1).

$$= \frac{x^4}{4} \log x - \frac{x^4}{16} + C$$

Example 65: $I = \int x^3 \sin x \, dx$

It will be convenient to use x^3 as the first term; as the differential of the same ultimately reduces to zero.

Example 66: $y + 2y'^{2} = 7 - 6x + 11x^{2}$

if $y(1) = 4$

The boundary condition is not given at the origin, so we expand y as a power series in (x-1). Therefore given equation can also be written as

$$y + 2y'^{2} = 12 + 16(x-1) + 11(x-1)^{2}, \text{ where } y(1) = 4$$

Assuming that

$$y = \overset{4}{a} + b(x-1) + c(x-1)^{2} + d(x-1)^{3} + e(x-1)^{4} + f(x-1)^{5} + ---$$

$$y' = b + 2c(x-1) + 3d(x-1)^{2} + 4e(x-1)^{3} + 5f(x-1)^{4} + ----$$

Using boundary condition,

$a = 4$; Further comparing coefficients:

Absolute term	Terms of (x-1)
$a + 2b^{2} = 12$	$b + 2*2*2cb = 16$
$4 + 2b = 12$	$b(1 + 8c) = 16$
$2b^{2} = 8$	Since b has 2 values, therefore
$b^{2} = 4$	either $c = 7/8$
$b = \pm 2$	or $c = -9/8$

Terms of $(x-1)^{2}$

$$c + 2\ [\ 2*3bd + 4c^{2}\ ] = 11$$

$$c + 12bd + 8c^{2} = 11$$

$$\frac{7}{8} + 24d + \frac{49}{8} = 1$$

$$d = 1/6$$

Similarly three more values of d shall be obtained using three remaining combination of the values of b & c. One solution is

$$y = 4 + 2(x-1) + \frac{7}{8}(x-1)^2 + \frac{1}{6}(x-1)^3 + ----$$

INTEGRAL EQUATIONS

Similarly we can solve the integral equation and integro differential equations.

Example 67: Solve $\int_0^x\int_0^x ydx.dx + 2\int_0^x y\,dx = \frac{x^3}{2} + 5x^2 + 8x$

Assume $y = a^4 + b^3x + c^0x^2 + d^0x^3 + ex^4 + ----$

Integrating

$$\int_0^x ydx = a^4x + \frac{b^{3/2}}{2}x^2 + \frac{c^0}{3}x^3 + \frac{d^0}{4}x^4 + ----$$

$$\int_0^x\int_0^x ydx = \frac{a^2x^2}{2} + \frac{b^{1/2}x^3}{6} + \frac{c^0x^4}{12} + ----$$

Coefficient of	Equation	
x	$2a = 8$	; $a = 4$
x^2	$2.\frac{b}{2} + \frac{a}{2} = 5$	; $b = 3$
x^3	$2.\frac{c}{3} + \frac{b}{6} = \frac{1}{2}$	; $c = 0$
x^4	$2.\frac{d}{4} + \frac{c}{12} = 0$	; $d = 0$

Hence exact solution is

$$y = 4 + 3x$$

Similarly we can solve even the non linear integro-differential equations.

Example 68: Solve $2y^2 + (y')^2 + y\int_0^x y\,dx = 18 + 9x + 40x^2 + 8x^3 + 8x^4 + \frac{4}{3}x^5$.

where $y(0) = 3$; $y'0) = 0$.

Assuming

$$y = a + \overset{0}{b}x + cx^2 \overset{0}{+} dx^3 + ex^4 + fx^5 + gx^6 + ---$$

$$y' = \overset{0}{b} + 2cx + 3dx^2 + 4ex^3 + 5fx^4 + 6gx^5 + 7hx^6 + ---$$

& $$\int_0^x y\,dx = ax + \frac{b}{2}x^2 + \frac{c}{3}x^3 + \frac{d}{4}x^4 + \frac{e}{5}x^5 + \frac{f}{6}x^6 + ---$$

Boundary conditions lead to a = 3 and b = 0

Coefficient	Equation
Constant	$2*a^2 + b^2 = 18$; Okay
x	$2*2ab + 2b*2c + a^2 = 9$; Okay
x^2	$2*2ac + 2b^2 + 2*3bd + 4c^2 + ab + \frac{ab}{2} = 40$
OR	$12c + 0 + 0 + 4c^2 = 40$; c= 2(OR -5)

Case 1 : Taking c = 2

x^3	$2*2ad + 2*2bc + 2*4be + 2*6cd + \frac{ac}{3} + \frac{b^2}{2} + ac$ $= 8$ OR $12d + 0 + 0 + 24d + 2 + 0 + 6 = 8$; OR d = 0
x^4	$2*2ae + 2c^2 + 2*8ce = 8$; $12e + 8 + 32e = 8$; e = 0
x^5	$2*2af + 2*10cf + \frac{c^2}{3} = 4/3$; OR f = 0

Hence one solution is $y = 3 + 2x^2$.

TRANSCENDENTAL EQUATIONS AND PARTIAL DIFFERENTIAL EQUATIONS

Even the equations of these types have been successfully solved using the Urdhava Sutra, other Vedic sutras and the series solution method. The details are available elsewhere (Ref. 10).

Example
$$\int_0^x y\,dx + 2\left(\frac{dy}{dx}\right)^2 = 2x + 8x^2 + 7x^3 + \frac{9}{4}x^4$$

If $y(0) = 2$ & $y'(0) = 0$

OPINIONS OF THE PARTICIPANTS

Everybody who has learnt a little about the Vedic Mathematics has been thrilled by the superfast, almost magic speed calculation procedures of the Vedic Mathematics.

The Spiritual Study Group at Roorkee has been organising, Elementary and Advanced Course on Vedic Mathematics since May, 85. All the participants are asked to fill up the response forms comparing the present system of Mathematics with the Vedic system. A total of 11 questions were asked. The minimum score in favour of Vedic Mathematics for any question in any course so far was 80%. Most of the question got 100% favour. Further, comments of participants are motivating and inspiring for spreading Vedic Mathematics and its systematic teaching in schools and colleges.

A few glimpses of the participants opinions :-

1. A boon for all competitive examinations.
- Could never imagine it would be so interesting.
- Very pleasant to have this course at the university.
- How advanced our ancestors were even in Mathematics.
- If Vedic Maths was taught in every School, computers would have developed in India.
- Excellent course.
- Want to learn advanced topics.
- Increases devotion towards our INDIA and its ancient culture.
- Now I wish to learn Sanskrit to know more about our ancient scientific developments.
- Every person on earth should have the knowledge of Vedic Mathematics.
- It was a boon to learn Vedic Mathematics in Dushera holidays (I had planned for my regular studies).
- Want to learn more about Vedic Mathematics like factorisation etc., which are difficult in modern system.

- Amazing speed, very wide applications covering almost all the mathematical operations.
- A dynamic step for its introduction in educational system is a must.
- Has created an interest to know more about it.
- It can be of great use for children if made part of syllabus.
- Mysterious nature of numbers. After little practice problems can be mentally solved at magic speed.
- Very educative, has much scope for coming generation to start its practice from beginning.
- It is a strong proof of our rich cultural heritage.
- By Vedic methods maths which is a burden, be-comes a joy for the children.
- This is the most interesting and thrilling time of my life. Being a mathematics teacher I have interest in maths but this Vedic Mathematics has thrilled me. When I came in contact with the Vedic Mathematics, it gave such a pleasure, which I can't explain in words.

A teacher of Adarsh Bal Niketan

- The lectures were eye openers and most thrilling.
- A glimpse of the lofty trekking in Himalayas of ancient knowledge in Maths, developed with spiritualism. Cudos to SSG.

A Professor of University

- I have enjoyed all aspects of Vedic Maths course - beautifully blissful two weeks.

Mrs. R. Leahy
Iowa U.S.A.

- A nery enjoyable course - looking forward to learning deeper applications.

S. Olvsson
Sweden

- I enjoying practicing Vedic Maths as much as I enjoy Meditating.

R. Kneedler

DISCUSSIONS

The essence of Vedic system is that it is simple, direct, one line and mental. The Vedic sutras are applicable to a wide variety of problems. They are very easy to understand and a delight to use.

1. The property of numbers is very extensively exploited in the Vedic Mathematics particularly in the field of computations. The Vedic approach has many special methods - indeed many more of them than conventional methods. This large flexibility of methods finds itself reflected in the mind, when approaching problem from Vedic view point. This indeed is the great benefit of the approach. One wonders at the reason of the supreme simplicity and ease of the Vedic method, which is really lacking in most of our usual methods.

2. Vedic maths is perfectly adaptable in oral teaching and mental calculation. They of course demand regular practice of simple problems. They are perfectly in tune with the oral Vedic tradition suplemented by terse statements of important points-called Sutras, which act as an aid to the memory.

3. Even a little exposure to the Vedic maths approach coupled with some practice clearly shows us that we are dealing with a entirely new and direct way of thinking.

4. In May 1985 when the first Vedic Mathematics course was being organised at the University campus, one of our students from Nepal, Sh. Bodhswarup (of B.E. Final Year Civil) who had just heard about the Vedic Mathematics book by Shankaracharya Ji Maharaj knew about the Vedic general method of multiplication (based on the Urdhava sutra). He had a perfect mastery over this method. He also introduced to us the Sudha upsutra and its use for Vedic method of addition subtraction, and compound addition and subtraction. Mr. Bodh had learnt these Vedic methods from his spiritual Guru who also was not aware of the Shankaracharya Ji's publication.

Further, Dr. Khanna of B.H.U. also informed us about his working knowledge of the general Vedic method of multiplication.

Dr. Ram Chandra from Jodhpur, who was not aware of the Shankaracharya Ji's publication, explained to us the Vedic Method of checking the results of any multiplication. Later on we found the procedure to be the straight application of the Gunit Sutra (no. 15).

In Bhaskaracharyas Leelavati, one of the methods of multiplication uses the deviation of the number from the base, which is very close to the Nikhilam method of Vedic Maths.

The alpha numeric coding system is based on the verse called Katadi (कटादि) sutra in ancient "Phalit Jyotish" (फलित ज्यौतिष) - Predictive astrology. There are several applications of the same. For example one of the matrices of (size 8x12) 96 elements giving the relative weightage, due to the inter planetry positions, has been codified in the simple four line Sanskrit verse. All the 96 elements of this matrix can be obtained at any time by simply decoding the verse.

Further, Dr. Rahul from Gorakhpur has recently decoded the Gayatri using the same key. The first part of the Shatakshara, Gayatri Mantra has 24 letters. The numerical value of thus adds up to the auspicious number 108. Many persons are using Vedic addition process with slight variations.

All these indicate clearly that the different facets of Vedic Mathematics have been scattered and shattered due to mass destruction and killings at the ancient Vedic seats of learning. There is a need to do further extensive research both in applications and cross linkage. But, whatever is available today is sufficient to make an excellent start both at the teaching as well as at research level.

5. Vedic Mathematics, due to its multi choice facility, provides wonderful opportunities for the development of the innovative and research faculty of the young students. A large number of Vedic Maths participants have been coming forward with very interesting and useful links. The age and qualification is no barrier in this process. The extensions of the Nikhilam method for multiplying three or more numbers and the use of different bases and subbase have all been provided by one of our UG students.

It shall be interesting to point out the recent research work of Sh. Sanjay, who has just obtained his Engineering Diploma. Mr. Sanjay has very successfully extended, the Vedic method of cubing, to find any power of a given number even beyond the computer capability. Further, he has found that the Urdhava method of solving linear simultaneous equations can be used for solving for even 8 or 10 unknowns using the Pivoting technique. The whole working can be completed in just 2 pages.

6. The Vedic Maths is primarily for mental working and has the potential of developing the human computer to wonderful level where the results of the problems flow very naturally, with the least amount of effort, in the typical Vedic way. This approach provides a corrective methodology to the problem of mental slavery to the calculators.

Further, Vedic algorithms have considerable potentials for automatic computations and small computers can handle much bigger problems due to the ease & simplicity of processes and lesser memory requirements.

Let us learn, teach and preach Vedic Mathematics which children learn with smiles on their face and joy in their hearts.

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